GEOMETRIC CONSTRUCTION OF THE CAUSTIC SURFACE OF CATADIOPTRIC NON CENTRAL SENSORS

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Abstract Most of the catadioptric cameras rely on the single viewpoint constraint that is hardly fulfilled. There exists many works on non single viewpoint catadioptric sensors satisfying specific resolutions. In such configurations, the computation of the caustic curve becomes essential. Existing solutions are unfortunately too specific to a class of curves and need heavy computation load. This paper presents a flexible geometric construction of the caustic curve of a catadioptric sensor. Its extension to the 3D case is possible if some geometric constraints are satisfied. This introduces the necessity of calibration that will be briefly exposed. Tests and experimental results illustrate the possibilities of the method.

Acknowledgments

We would like to thank Professor P.J. Giblin for his relevant advices and help and Mr F.Richard for providing us some test materials.

Introduction

The caustic curves are an optical phenomenon studied since Huygens and Hamilton [Hamilton, 1828]. They are enveloppes of the reflected or diffracted light. Most of the existing vision systems are designed in order to achieve the convergence of the incident rays of light at a single point called ‘effective viewpoint’.
Such a configuration of sensors can be seen as a degenerated form of the caustic reduced to a single point. The catadioptric sensors are divided into two categories, the ones fullfilling the single viewpoint constraint (SVC) where the caustic is reduced to a point and the none SVC that need the computation of the caustic. The single viewpoint constraint [Rees, 1970, K. Yamazawa, 1993, Nalwa, 1996, Nayar, 1997, V. Peri, 1997, Baker and Nayar, 1998, J. Gluckman, 1999] provides easier geometric systems and allows the generation of correct perspective images. However it requires a high precision assembly of the devices that is hardly fulfilled practically [Fabrizio et al., 2002] and it also faces the problem of uniformity of resolution. The problem of designing a catadioptric sensor that results in improved uniformity of resolution compared to the conventionnal sensor has been studied by several approaches [J.S. Chahl, 1997, Backstein and Padjla, 2001, Conroy and Moore, 1999, S. Gaetcher, 2001, R.A. Hicks, 2000, R. A. Hicks, 2001, M. Ollis, 1999]. These solutions rely on the resolution of differential equations that are in most cases solved numerically providing a set of sampled points.

There are many ways to compute the caustic of a smooth curve, generally they are too specific to a finite class of curves and/or to a particular position of the light source [Bellver-Cebreros et al., 1994]. More complex methods based on the illumination computation (the flux-flow model) are studied in geometric optics [Burkhard and Shealy, 1973]. They are highly applied in computer graphics when a realistic scene rendering is required [Mitchell and Hanrahan, 1992, Jensen, 1996]. A method for determining the locus of the caustic is derived from this flux-flow model, analysis and tests are carried out on conic shape mirrors in [Swaminathan et al., 2001], in order to extract the optical properties of the sensor.

In this paper we study a method allowing the computation of the caustic based on a simple geometric constructions as related in [Bruce et al., 1981]. The interest of the approach is its moderate computational load and its great flexibility toward unspecified smooth curves. We will show that its extension to the third dimension is possible if we consider the problem as a planar one in the incident plane and by assuming that the surface has a symmetry axis. We will show that the geometric construction can be applied if the light source is placed on this axis. This highlights the problem of ensuring the alignment between the camera and the mirror and points out the importance of a robust and accurate calibration that will be briefly introduced. Finally experimental results carried out on analytically defined mirrors and non explicit equation mirrors are presented.

1. The caustic curve: definition and construction

The catadioptric sensor that does not comply with the single viewpoint constraint, require the knowledge of the caustic surface if one expects to calibrate it. Defined as the enveloppe of the reflected rays, the caustic curve gives the direction of any incident ray captured by the camera.
In this section, we present in detail two methods applied to the caustic curve computation for systems combining a mirror and a linear camera. The first method derives from the flux-flow computation detailed in [Burkhard and Shealy, 1973]. [Swaminathan et al., 2001] used this technique on conical based catadioptric sensors. A detailed analysis and relevant results are obtained.
The second method is based only on geometrical properties of the mirror curve. Caustic surface point is determined by approximating localy the curve by a conic where both the light source and the caustic point are foci of this conic.

1.1 Flux-flow model

The vanishing constraint on the Jacobian is applied to the whole class of conical mirrors. Though it can be applied for any regular curves, work exemples exposed here are smooth curves discussed in [Swaminathan et al., 2001].

We define $\mathbf{N}$, $\mathbf{V_i}$ and $\mathbf{V_r}$ as respectively the normal, the incident and the reflected unit vectors, at the point P of the mirror M. The three vectors are functions of the point P, then if M is parametrised by t, they are functions of t.
According to the reflection laws, we have:

$$\mathbf{V_r} - \mathbf{V_i} = 2(\mathbf{V_r}.\mathbf{N})\mathbf{N} \Rightarrow \mathbf{V_i} = \mathbf{V_r} - 2(\mathbf{V_r}.\mathbf{N})\mathbf{N}$$

Assuming the point P to be the point of reflection on M, if we set P_c as the associated caustic point then P_c satisfies:

$$P_c = P + r\mathbf{V_i}$$

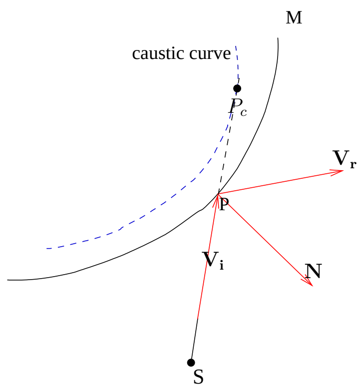

Figure 1. Catacaustic or caustic by reflection: the dashed curve shows the locus of the reflected rays enveloppe.

r is a parameter and since P and $\mathbf{V_i}$ depend on t, P_c is a function of (t, r). If the parametric equation of M is given as:

$$M : \begin{cases} z(t) \\ \gamma(t) \end{cases}$$

Then, the the jacobian of P_c is:

$$J(P_c) = \begin{vmatrix} \frac{\partial P_z}{\partial t} & \frac{\partial P_z}{\partial r} \\ \\ \frac{\partial P_\gamma}{\partial t} & \frac{\partial P_\gamma}{\partial r} \end{vmatrix} = \begin{vmatrix} \dot{P}_z + r\dot{V}_{iz} & V_{iz} \\ \dot{P}_\gamma + r\dot{V}_{i\gamma} & V_{i\gamma} \end{vmatrix} \tag{1}$$

$J(P_c)$ must vanish, thus we solve the equation $J(P_c) = 0$ for r:

$$r = \frac{\dot{P}_z V_{i\gamma} - \dot{P}_\gamma V_{iz}}{V_{iz}\dot{V}_{i\gamma} - V_{i\gamma}\dot{V}_{iz}} \tag{2}$$

Obviously r is a function of t. Then we can have the parametrised form of the caustic.
In the case of conical curve M, we can parametrise it by t, according to the following form:

$$M : \begin{cases} z(t) = t \\ \gamma(t) = \sqrt{C - Bt - At^2} \end{cases}$$

Where A,B and C are constant parameters. The implicit equation of M can be deduced:

$$f(x, y) = Az^2 + \gamma^2 + Bz - C = 0$$

Explicit solutions and details for these curves can be found in [Swaminathan et al., 2001]. The same method is extended to three dimensionnal curves. M is then a smooth surface and its caustic surface relatively to the light source can be determined by solving a three by three matrix determinant.

. **Remark1:** Parametrized equation of M is known
Assuming the profile of M known, one can expect an important preprocessing step. First because of the computation of the Jacobian then the resolution for r according to the vanishing constraint.
Analytical resolution of r will provide exact solution for the caustic. However, if equation $J(P_c) = 0$ can be solved for conical class curves, we are not always able to solve it analyticaly for any smooth curves.

. **Remark 2:** The profile of the mirror is not known
This is the most general case we have to face. M is given by a set of points, then the computation of r must be numerical. This can be difficult to handle with, especialy when we extend the problem to three dimension curves where r is root of a quadratic equation.

Bearing in mind the advantages and weaknesses of the Jacobian method, we present here another technique to compute the caustic curve, where only local mathematical properties of M are taken into account.
The caustic of a curve M, is function of the source light S. The basic idea is to consider a conic where S is placed on one of its foci. A simple physical consideration shows that any ray emitted by S should converge on the other focus F. It is proved in [Bruce et al., 1981] that for any P on M, there is only one caustic with properties mentioned above so that F is the caustic point of M, relative to S, at P.
A detailed geometric construction of F will be described here, first for plane curves then followed by an extension to three dimensional ones.

1.2 Geometrical construction

DEFINITION 1 *Considering a regular(i.e smooth) curve M, a light source S and a point P of M, we construct Q as the symetric of S, relative to the tangent to M at P. The line (QP) is the reflected ray.(see Fig. 2)*

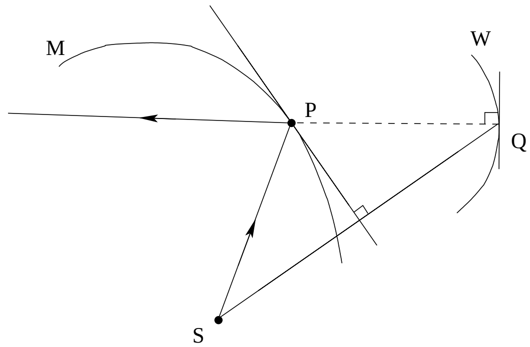

Figure 2. Illustration of the orthotomic W of the curve M, relative to S. W is interpreted as the wavefront of the reflected wave.

When P describes M, Q describes a curve W where (QP) is normal to it at Q. W is known as the orthotomic of M, relatively to S. The physical interpretation of W is the wavefront of the reflected wave.
It is equivalent to define the caustic curve C of M, relative to S as:

- *The evolute of W i.e. the locus of its centers of curvature [Rutter, 2000].*
- *The enveloppe of the reflected rays.*

DEFINITION 2 *Given two regular curves f and g of class C^n, with a common tangent at a common point P, taken as $(0\ 0)^t$ and the abscissa axis as the tangent. Then this point is an n-order point of contact if:*

$$\begin{cases} f^{(k)}(0) = g^{(k)}(0) = 0 \quad \textit{if} \quad 0 \leq k < 2 \\ f^{(k)}(0) = g^{(k)}(0) \quad \textit{if} \quad 2 \leq k \leq n-1 \\ f^{(n)}(0) \neq g^{(n)}(0) \end{cases}$$

There is only one conic $\mathcal{C}$ of at least a 3-point contact with M at P, where S and F are the foci. F is the caustic point of M at P, with respect to S.
For the smooth curve M, we consider the parametrised form:

$$M : \begin{cases} x = t \\ y = f(t) \end{cases} \tag{3}$$

where $P = (x\ y)^t \in M$. The curvature of a M at P is given by:

$$k = \frac{|P^{'}P^{''}|}{|P|^3} = \frac{f^{''}(t)}{\sqrt{1 + f^{'}(t)^2}^3} \tag{4}$$

with

$$|P^{'}P^{''}| = \begin{vmatrix} x^{'} & x^{''} \\ y^{'} & y^{''} \end{vmatrix}$$

With regard to these definitions, we can deduce that M and $\mathcal{C}$ have the same curvature at P. If k is known, we are able to build the caustic C independendly of W.
For more details and proofs of these affirmations, reader should refer to [Bruce et al., 1981]. We give here the geometrical construction of the focus F, with respect to the conic $\mathcal{C}$ complying with the properties decribed above. Fig. 3 illustrates the geometrical construction detailed below.

- Compute O, center of curvature of M at P, according to $r = \frac{1}{k}$, radius of curvature at P. O satisfies:

$$O = P + |r|\mathbf{N} \tag{5}$$

- Project orthogonaly O to (SP) at u.
  Project orthogonaly u to (PO) at v.
  (Sv) is the principal axis of C.
- Place F on (Sv) so that (OP) is bisectrix of $\widehat{SPF}$.

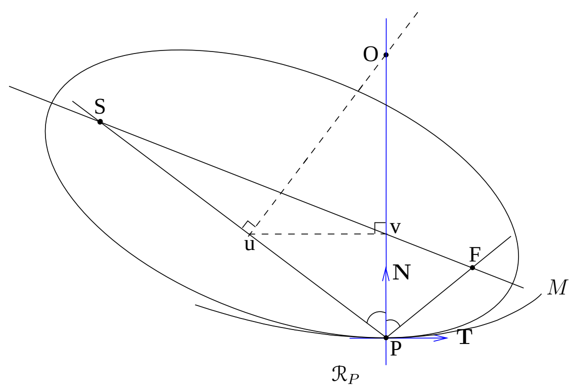

Figure 3. Geometric construction of the caustic point expressed in the local Frenet's coordinates system $\mathcal{R}_\mathcal{P}$

Depending on the value taken by k, $\mathcal{C}$ can be an ellipse, a hyperbola or a parabola. For the first two, F is at finite distance of S ($\mathcal{C}$ has a center) and $k \neq 0$. If $k = 0$, F is at infinity and $\mathcal{C}$ is a parabola.
We consider here only the case where S is at a finite distance from M. If S is placed at infinity or projected through a telecentric camera, the incident rays are parallel and the definition of the caustic is slightly different.

It is more simple to express the curves using the local Frenet's coordinates system at P and denoted $\mathcal{R}_P$. Hence P is the origin of $\mathcal{R}_P$ and we have $O = (0\ |r|)^t$ since $\mathbf{N}$ is the direction vector of the ordinate axis. One can easily prove that the generic implicit equation of $\mathcal{C}$ in $\mathcal{R}_P$ is

$$ax^2 + by^2 + 2hxy + y = 0 \tag{6}$$

and that the curvature is $k = -\frac{1}{2a}$. However, it is obvious to see that the construction of F does not require the computation of the parameters of (6).

We can write down the coordinates of F in R_p if we express analytically the cartesian equation of each line of Fig. 3.

$$\mathcal{C} : \begin{cases} x_f = -\frac{y_s^2 x_s |r|}{2y_s(x_s^2+y_s^2)-y_s^2|r|} \\ \\ y_f = \frac{y_s^2|r|}{2(x_s^2+y_s^2)-y_s|r|} \end{cases} \tag{7}$$

The generic expression of the coordinates of F depend only on the source S and the curve M through r.

1.3 Extension to the third dimension

Given a three dimension surface $\mathcal{M}$, we decompose it into planar curves M which are the intersections of $\mathcal{M}$ with the incident planes. According to the Snell's law of reflection, the incident and the reflected rays are coplanar and define the plane of incidence Π_P. Since the caustic point associated to S and the point P belongs to (PQ) (see Fig.1.2), can we expect to apply the geometric construction to M? (See Fig. 4 for a general illustration.)
A problem could arise if we consider a curve generated by a plane containing the incident ray and intersecting the mirror, then the normals to the generated curve may not be the normal to the surface, the computed rays are then not the reflected ones.

We will now apply the construction on $\mathcal{M}$ a surface that has a revolution axis with S lying on it (see Fig. 5).
Step 1: $(\Omega z) \in \Pi_P$ with respect to $S \in (\Omega z)$.
Given the standard parametrization of the surface of revolution $\mathcal{M}$, expressed in an arbitrary orthogonal basis $E = (\Omega, \mathbf{x}, \mathbf{y}, \mathbf{z})$ such that (Ωz) is the revolution axis of $\mathcal{M}$:

$$\mathcal{M} : \begin{cases} x(t,\theta) = r(t)\cos\theta \\ y(t,\theta) = r(t)\sin\theta \\ z(t,\theta) = k(t) \end{cases} \tag{8}$$

The normal unit vector to $\mathcal{M}$ at $P = \begin{pmatrix} x & y & z \end{pmatrix}^t$ can be defined as:

$$\mathbf{N} = \frac{\mathbf{A} \wedge \mathbf{B}}{|\mathbf{A} \wedge \mathbf{B}|} \text{ where } \wedge \text{ is the cross product, } \mathbf{A} = \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} x_\theta \\ y_\theta \\ z_\theta \end{pmatrix}$$

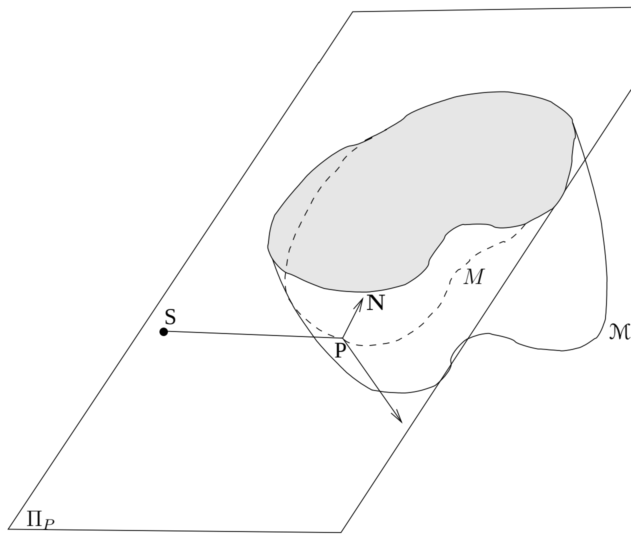

Figure 4. Generic case of a three dimension curve: can it be decomposed into planes and solved as planar curves for each point P of M?

and the subscripts t and θ refere to the partial derivatives with respect to t and θ.

Thus,

$$\mathbf{N} = \frac{1}{|\mathbf{A} \wedge \mathbf{B}|} \begin{pmatrix} r^{'} \cos\theta \\ r^{'} \sin\theta \\ k^{'} \end{pmatrix} \wedge \begin{pmatrix} -r\sin\theta \\ r\cos\theta \\ 0 \end{pmatrix} = \frac{1}{\sqrt{r^{'2} + k^{'2}}} \begin{pmatrix} -k^{'} \cos\theta \\ -k^{'} \sin\theta \\ r^{'} \end{pmatrix}.$$

Let us consider now the rotation along (Ωz), given by the rotation matrix:

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

if the rotation angle is assumed to be θ.
We then define $B = (\Omega, \mathbf{u}, \mathbf{v}, \mathbf{z})$ as the orthogonal coordinates system obtained by applying R to E. The coordinates of $\mathbf{N}$ in B is:

$$R.\mathbf{N} = \frac{1}{\sqrt{r^{'2} + k^{'2}}} R \begin{pmatrix} -k^{'} \cos\theta \\ -k^{'} \sin\theta \\ r^{'} \end{pmatrix} = \frac{1}{\sqrt{r^{'2} + k^{'2}}} \begin{pmatrix} -k^{'} \\ 0 \\ r^{'} \end{pmatrix} \qquad (9)$$

$\mathbf{N}$ has a null component along $\mathbf{v}$, hence the line $(P, \mathbf{N})$ belongs to the plane $\Pi = (\Omega, \mathbf{u}, \mathbf{z})$. Moreover, since $S \in (\Omega z)$, one can deduce that $\Pi = (S, (P, \mathbf{N})) =$

Π_P.

Step 2: $\mathbf{N} = \mathbf{n}$

With respect to the hypotheses made above, we compute $\mathbf{n}$ according to the parametric equation of M, expressed in the coordinate system (Ω, u, z):

$$M : \begin{cases} u = r(t) \\ z = k(t) \end{cases} \tag{10}$$

Thus the tangent to M at P is defined as $T = \begin{pmatrix} r' \\ k' \end{pmatrix}$ end the unit normal vector is:

$$\mathbf{n} = \frac{1}{\sqrt{r'^2 + k'^2}} \begin{pmatrix} -k' \\ r' \end{pmatrix} \tag{11}$$

By combining Eq. (9) and Eq. (11), we have the equality $\mathbf{N} = \mathbf{n}$.

This proves that in particular configurations that involve an on-axis reflection, N is normal to $\mathcal{M}$ and to M at P, then the geometric construction holds.

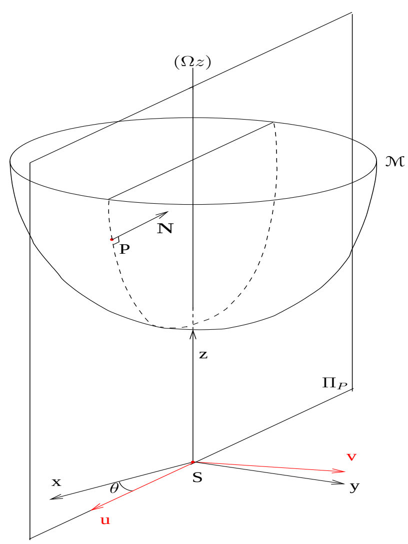

Figure 5. If the source light S is placed on the revolution axis, the geometric construction can be applied on each slices of incident planes.

2. Ensuring the alignment Mirror/Camera: catadioptric calibration

Most of the catadioptric sensors rely on a mirror having a surface of revolution. In general most of the applications assume the perfect alignment between the optical axis of the camera and the revolution axis of the reflector. As shown in the previous sections, the perfect alignment between the two axes introduce a simplification in the computation of the caustic surface. We may wonder how realistic is this condition and if it still holds in the real case? It then appears the necessity of an accurate and robust calibration procedure to retrieve the real position of the mirror with respect to the camera.
Calibration in general relies on the use of a calibration pattern to ensure a known structure or metric in the scene. Due to the non linear geometry of catadioptric sensors, the computation of the parameters (position camera/mirror, intrinsics of the camera, ...) can turn into a major non linear problem. Previous calibration works are not numerous and are in general connected to the shape of the mirror [Geyer and K.Daniilidis, 2002, Cauchois et al., 2000].

A much simpler approach would be to consider the mirror as a calibration pattern. The mirror is generally manufactured with great care (precision less than 1 micron) and its shape and surface are perfectly known. Using the mirror as a calibration pattern avoids the non linearity and turns the calibration problem into a linear one. The basic idea is to assume the surface parameters of the mirror as known and to use the boudaries of the mirror as a calibration pattern [Fabrizio et al., 2002]. As a major consequence the calibration becomes robust as the mirror is always visible, the calibration is then independent from the content of the scene and can be performed anytime needed. The calibration relies on one or two homographic mapping (according to the design of the mirror) between the mirror borders and their corresponding images. To illustrate this idea let us consider a catadioptric sensor developped by VSTONE [VstoneCorp,] that has an interesting property. A little black needle is mounted at the bottom of the mirror to avoid unexpected reflections. The calibration method is based on the principle used in [Gremban et al., 1988]. This approach is known as the two grid calibration. Two different 3D parallel planes P_1 and P_2 are required (see Fig.6). The circular sections of the lower and upper mirror boundaries C_1 and C_2 are respectively projected as ellipses E_1 and E_2. Homographies H_1 and H_2 are estimated using the correspondence between C_1/E_1 and C_2/E_2. The distance between the two parallel planes P_1 and P_2 respectively containing C_1 and C_2 being perfectly known, the position of the focal point is then computed using both H_1 and H_2 by back projecting a set of n image points on each plane. In a second stage the pose of the camera is estimated. We then have the complete pose parameters between the mirror and the camera and the instrinsics parameters of the camera. The reader may refer to [Fabrizio et al., 2002] for a complete overview of this method.

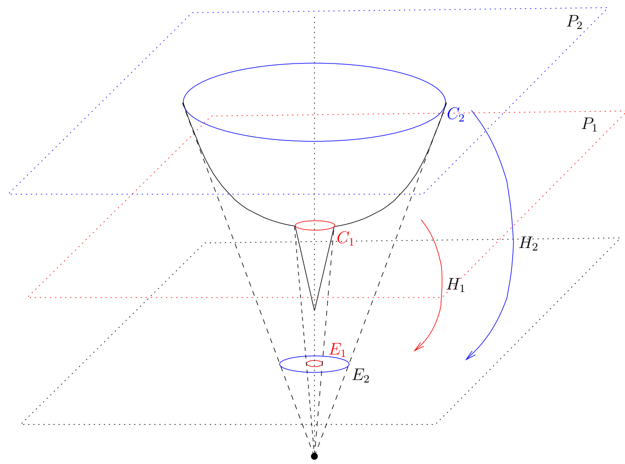

Figure 6. Calibration using parallel planes corresponding to two circular sections of the mirror

The same idea can be used if only one plane is available. In that case only the image E_2 of the upper boundary of the mirror C_2 is available. The homography H_2 can then be estimated. There is a projective relation between the image plane and the plane P_2. The classic perspective projection matrix is $P = K(R \mid t)$ with K the matrix containing the intrinsics and R,t the extrinsics. The correspondence between E_2 and C_2 allows an identification of P with H_2. The only "scene" points available for calibration all belong to the same plane P_2. P can then be reduced to the following form $P = K(r_1\ r_2\ t)$, where $r_1 r_2$ correspond to the first two columns vectors of the rotation matrix R. As a consequence the matrix $H_2 = (h_{21}\ h_{22}\ h_{23})$ can be identified with $P = K(r_1\ r_2\ t)$ giving :

$$(h_{21}\ h_{22}\ h_{23}) \sim K(r_1\ r_2\ t) \tag{12}$$

The matrix H_2 is constrained to be a perspective transform by the rotation R giving the two following relations :

$$\begin{array}{l} h_1^T K^{-T} K^T h_2 = 0 \\ h_1^T K^{-T} K^T h_1 = h_2^T K^{-T} K^T h_2 = 0 \end{array} \tag{13}$$

If an estimate of K is available it becomes possible to compute R and t using Eq. (12). The reader may refer to [Zhang, 2000, P.Sturm, 2000] for a complete overview of this computation.

The two presented approaches allow a full calibration of catadioptric sensors and can ensure if the geometry of the sensor fulfills the desired alignment between the camera and the mirror. It becomes then possible to choose the adequate computation method. These methods are not connected to the shape of the mirror and can then be applied to all catadioptric sensors. It is interesting to notice that catadioptric sensors have an interesting property as they carry their own calibration pattern.

3. Experimental results

The geometrical construction is illustrated here with smooth curves exemples. Their profiles are defined by the parametrised equations. As we can see in section 2, only Eq. (4) is specific to M, the curvature at P implies only the first two derivatives at P with respect to t. Hence, if the profile of M is given only as a set of sampled points, the algorithm can handle it if the sampling step is small enough.

3.1 Plane curves

- Exemple 1:
  Let M be the conic defined by its parametrised and implicit equations:

$$M : \begin{cases} x(t) = t \\ y(t) = b\sqrt{1 + \frac{t^2}{a^2}} - c \end{cases} \tag{14}$$

and

$$f(x, y) = \frac{(y - c)^2}{b^2} - \frac{x^2}{a^2} - 1 = 0 \tag{15}$$

The first and second derivatives with respect to the parameter t are:

$$M^{'} : \begin{cases} x^{'}(t) = 1 \\ y^{'}(t) = \frac{bt}{a^2\sqrt{1+\frac{t^2}{a^2}}} \end{cases} \tag{16}$$

$$M^{"} : \begin{cases} x^{"}(t) = 0 \\ y^{"}(t) = \frac{b}{a^2}\frac{1}{\left(1+\frac{t^2}{a^2}\right)^{\frac{3}{2}}} \end{cases} \tag{17}$$

Compute r at P according to (4):

$$r = \frac{1}{k} = \frac{\sqrt{a^2(a^2 + t^2) + b^2t^2}^3}{a^4b} \tag{18}$$

then change the coordinate system to the Frenet's local coordinate system at P for an easier construction of F.

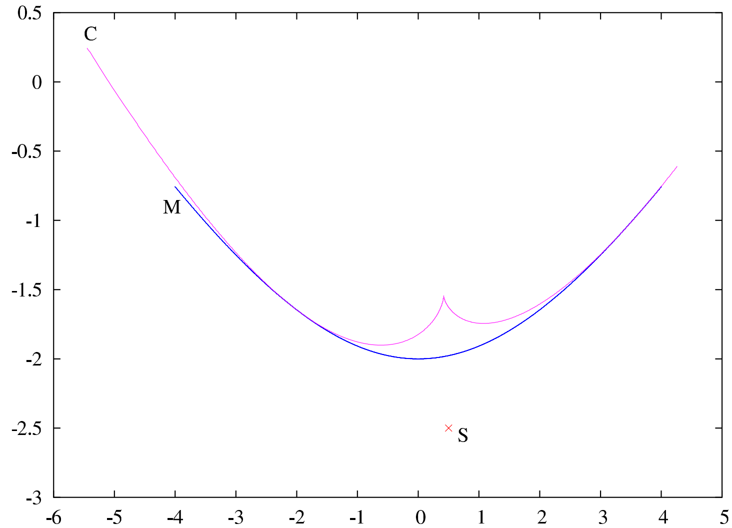

Figure 7. Caustic of a hyperbola for an off-axis reflection.

Fig. 7 shows the plot of the caustic for an off-axis reflection i.e. P is not on the symmetry axis of M. The parameters are $a = 4$, $b = 3$, and $c = 5$ and $S = [0.5\ 0.25]^t$.

- Exemple 2:
  This is the most general case we have to face: the reflector is given only by a set of sampled points, no explicit equation is known. The curvature at each point is numerically computed, providing a numerical estimation of the caustic.
  The mirror tested has a symmetric axis and the camera is placed arbitrarly on it. We computed the caustic curve relative to this configuration (see Fig. 8). The catadioptric sensor has been calibrated using a method similar to [Fabrizio et al., 2002], the sensor is then fully calibrated. Given a set of points taken from a scene captured by this sensor, we reproject the rays on the floor in order to check the validity of the method of construction. As illustrated in Fig. 9, the geometry of the calibration pattern is acurately reconstructed retrieving the actual metric (the tiles on the floor are squares of 30x30cm). The reconstruction shows that the farther we are from the center of the optical axis, the less acurate we are which is an expected result as the mirror was not computed to fulfill this property.

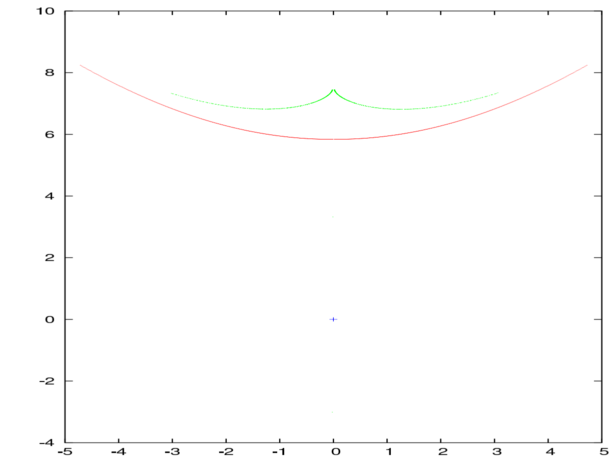

Figure 8. Caustic curve of the sampled mirror. The camera is placed at the origin of the coordinates system, represented by the cross.

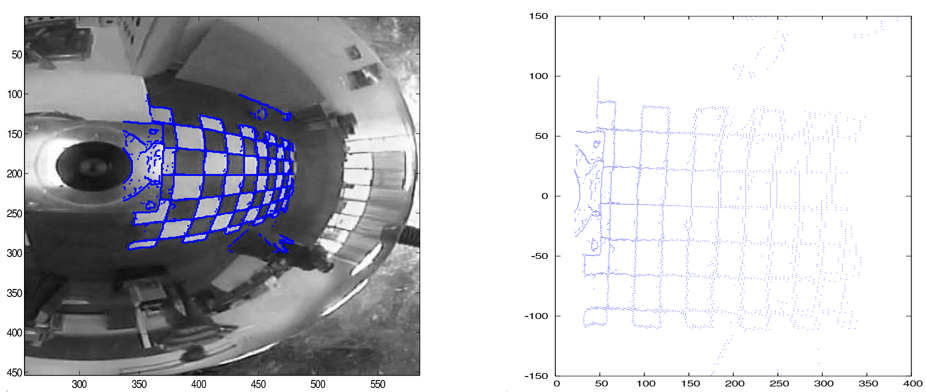

Figure 9. A scene captured by the sensor. The blue dots are scene points that are reprojected on the floor, illustrated on the left plot.

4. Conclusion

This paper presented a geometric construction of caustic curves in the framework of catadioptric cameras. When the single viewpoint constraint cannot be fulfilled, the caustic becomes essential if calibration and reconstruction are needed.

Existing methods imply heavy preprocessing work that can lead to an exact solution of the caustic if the mirror profile is known, however this is not guaranteed for general cases. The presented geometric construction is a very flexible computational approach as it relies only on local properties of the mirror. Since no special assumption is made on the mirror curve, except its smoothness, the presented work is able to solve cases of either known mirror profile or curves defined by a set of sample points fullfilling the aim of flexibility. The extension to 3D is possible under certain geometric restrictions.

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