

Influence of geometrical parameters on capillary forces

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Abstract—As miniaturization of objects and systems is further carried on, adhesion appears to be one major problem during the assembly and/or fabrication of micro-components.

This paper presents a model for the computation of capillary forces. For simple geometries, this model complies with literature results. In addition, it allows the computation of capillary force for non-axisymmetrical shapes. The complexity can arise from object shape (modelling for example an AFM tip) and/or from geometrical configuration. One very important result is the ability to compute the evolution of the capillary force depending on the tilt angle of the gripper with respect to the object. Using this results, it could be possible to manipulate small (a few tens of μm of characteristic dimension) objects with capillary condensation grippers.

Currently the model takes into account the contact angles, the relative humidity, temperature and the geometrical description of the problem. It is shown that it is possible to reach forces up to a few hundreds of nanonewton in magnitude.

This paper also presents a test bed developed in order to validate the models.

keywords : condensation, capillary forces, nanobridge, AFM

I. INTRODUCTION

One major problem occurring during the assembly of micro-components is adhesion, as quoted in literature [1], [2]. In everyday life, when picking up a macroscopic object, the major opposing force is gravity, which is a volume force.

When size diminishes and the objects are scaled down, surface forces become more and more important and the major opposing force to picking up and releasing parts becomes adhesion [3]. The force needed to separate two objects is also known as pull-off force. Adhesion can also prevent structures like RF-MEMS or any high aspect ratio structures from normal functioning [4].

The adhesion force is actually composed of different components : electrostatic force, van der Waals force, chemical forces, and capillary force. Electrostatic force can be avoided by properly choosing materials. Van der Waals force arises from the intrinsic constitution of matter : it is due to the presence of instantaneous dipoles. It becomes non-negligible under the nanometer scale. Chemical forces are due to the bondings between particles. It is active when objects are in contact (*i.e.* the distance between them is about an intermolecular distance).

Capillary force between two objects is due to the presence of liquid between them (see figure 1). It has already been shown [5] that it can be used to manipulate small objects

(300 – 500 μm of characteristic dimension) by manually placing a liquid droplet on the object (a fraction of μL). As capillary force is often the major component of adhesion, especially at high humidity levels, this paper aims at theoretically studying the feasibility of manipulating smaller objects (50 – 100 μm of characteristic dimension) using capillary condensation, *i.e.* without placing a droplet.

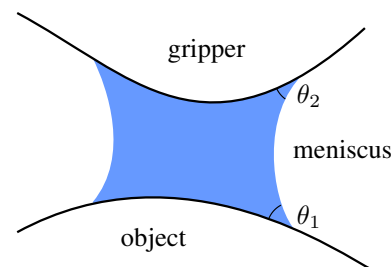


Fig. 1. Scheme of the problem : a gripper and an object are linked by a liquid meniscus, due to humidity condensation; the major non-geometrical parameters are the contact angles θ_1 and θ_2 and the surface tension γ_{lv} of the liquid-air interface (here the liquid is always water).

In assembly, it is as important to pick up as to release the object : the force applied onto the object should therefore be controllable. In macro-assembly, this problem does not exist : usual grippers can be closed (*i.e.* the part is locked between the digits of the gripper) or open (and the gripping force falls to zero). In micro-assembly, adhesion provides a minimal value for the "gripping" force. If the weight of the object is smaller than adhesion, the object cannot be released. Conversely, if adhesion is smaller than the object weight, the object cannot be picked up.

It is here proposed to control the force by tilting the tip (an application could be an AFM tip) with respect to the object. Models which assume objects and meniscus to be axially symmetrical [6], [7] therefore become unapplicable. A more general model has thus been developed to compute the capillary force without this constraint of axisymmetry.

This paper is organized as follows: first, the basic equations will be recalled (section II), and their validity discussed (section III). The model implemented in our simulation will then be presented (section IV). Results will then be explained (section V) and discussed (section VI) for different cases : first axisymmetrical shapes will be used for validation purpose only, then more complex configurations will be studied. Finally, a test bed used for validation purpose will be described.

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II. EQUATIONS

A. Introduction

This section will detail the different equations involved in the problem :

- the Kelvin equation [8], which rules the curvature of the liquid meniscus.
- the approaches encountered in literature to compute the capillary force, *i.e.* :
 - the Laplace approach, based on direct force calculations.
 - the energetical approach, based on the derivation of the total energy of the interfaces.

In all cases, gravity is neglected. This will be valid as long as the dimensions of the meniscus are a lot smaller than 1 mm.

B. Kelvin equation

In any method used to compute the capillary force, the meniscus shape appears explicitly or implicitly. In the Kelvin equation approach, it is involved through the total mean curvature H of the meniscus, or its inverse r , the mean curvature radius

$$H = \frac{1}{r} = \frac{1}{2r_K} = \frac{1}{2} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \quad (1)$$

where r_K is, in the case of capillary condensation, the so-called Kelvin radius, and r_1 and r_2 are the two principal curvature radii. The Kelvin radius is ruled by the Kelvin equation [3] which is the fundamental equation for capillary condensation. It links the curvature of the meniscus with environmental and materials properties.

$$r_K = \frac{\gamma V_m}{RT \log_e(p/p_0)} \quad (2)$$

where V_m is the molar volume of the liquid, R is the perfect gas constant (8.31 J/mol K), T the temperature (in kelvin) and p/p_0 is the relative humidity (RH), between 0 and 1. Typically, for water, this gives $r_K = 0.54 \text{ nm} / \log_e(RH)$, which gives for RH=90%, a Kelvin radius of about 5 nm at 20°C.

C. Laplace approach

With this method, the capillary force is splitted into two components, *i.e.* the Laplace force and the surface tension force [8].

It must be pointed out here that, in the general case, the force has no reason to be directed only along the axis of the distance between objects. It must be expressed as a vector. Actually, the expressions in this section implicitly assume that the meniscus (and the tip and the object) is axisymmetrical. Only one component is then different from zero. Consequently, this methodology cannot be applied in the general case, at least in the simple form presented here.

The Laplace force is due to the pressure difference $\Delta p = p_{in} - p_{out}$ across the liquid-vapor interface. Using the so-called Laplace equation, it can be expressed as [8]

$$F_L = -\Delta p A = -2\gamma H A^1 \quad (3)$$

where γ is the liquid-vapor interface surface tension, H is the total mean curvature of the meniscus and A is the area of the contact between the meniscus and the object. A is actually the effective area, *i.e.* the projection of the wetted interface onto a plane perpendicular to the force.

The other component of the capillary force is due to the surface tension of the liquid-object interface. Considering this surface tension as a constant

$$F_T = L \gamma \sin \theta \quad (4)$$

where L is the perimeter of the liquid-object interface, and θ the liquid object contact angle (this contact angle is supposed constant).

The capillary force is thus given by

$$F = -2\gamma H A + L \gamma \sin \theta \quad (5)$$

Note that F_T is always attractive, while F_L can be either attractive or repulsive depending on the sign of the mean curvature.

The calculation can also be made on the meniscus/tip interface, the result will obviously be the same.

Different approximations are usually made to this latter equation

- the assumption of axisymmetrical meniscus, that reduces the order of the equation.
- the limitation of the object shape.
- the *a priori* limitation to the shape of the meniscus : arc of circle, parabola,... [6], [9], [10]

Nevertheless, since the easiest way of controlling the force seems to be tilting one of the objects, the axisymmetry assumption does not hold anymore and the general equation has to be solved.

D. Energetical view

The same results can be obtained via an energetical approach. The total energy of the meniscus can be expressed as

$$W = \gamma_{LV} A_{LV} + \gamma_{LO} A_{LO} + \gamma_{LT} A_{LT} + \gamma_{OV} A_{OV} + \gamma_{TV} A_{TV} \quad (6)$$

where γ_{ij} is the surface tension of the i-j interface : **L**iquid, **V**apor, **O**bject and **T**ip, and A_{ij} are the areas of these interfaces. Note that here, the areas are the actual areas, and not the effective areas.

The capillary force in the z direction can then be computed using a classical derivative of the energy with respect to the distance between the objects. If the problem is axisymmetrical, the z direction is the axis of symmetry, otherwise it can be any direction along which the force is calculated².

¹note on the sign of the force : here, the force is positive if attractive.

²once again, a positive force means attraction

$$F = \frac{dW}{dz} \quad (7)$$

It can be shown that both formulations (Laplace approach and energetical approach) presented here are equivalent and obviously should lead to the same results. The model presented in this paper uses the energetical approach, as it will be explained later.

III. VALIDITY OF THE EQUATIONS

Both methods presented here are based on macroscopic hypothesis on the nature of the liquid and solids : matter is continuous and so are their properties such as surface tension. It has been shown experimentally that those models are valid down to meniscii of radii as small as 4 nm [11]. For smaller sizes, the discrete nature of matter should be taken into account, via molecular dynamics or Monte-Carlo calculations [12]. The results will thus have to be interpreted keeping in mind that their validity is not proven for very small sizes of meniscus (*i.e.* for meniscus with radii < 4nm).

Another discussion encountered in literature is the parameter kept constant during the derivative : should it be computed keeping the volume or the curvature constant [13] ? Different mechanisms play a role here, mainly the condensation/ evaporation rate. It seems admitted that the condensation takes place in the millisecond (< 5 ms [13]) scale while the evaporation needs more time. It has been measured meniscus stretching at tip-object distance much larger than r_K . Capillary condensation also has a long term (up to tens of days) component, which will not be considered here [14].

If experimental investigation could provide an estimation of the characteristic times involved in the processes, it would seem natural to compute the volume condensed -fulfilling the Kelvin equation- at the smallest tip-object distance and then compute the evolution of the force with constant volume when retracting the tip.

IV. MODEL AND SIMULATION SCHEME

It can be necessary to be able to cope with non axisymmetrical meniscii, for example to quantify the effect of geometrical difference with ideal case or to deal with complex shapes. As the equations in the general case cannot be solved analytically, a numerical model has been developed, which allows the user to define three dimensional shapes, with some limitations though, that are presented below.

The solver makes use of the software Surface Evolver (SE) [15]. The total energy of the meniscus is minimized, fulfilling the Kelvin equation on the shape of the meniscus. The energy is then derived (by finite differentiation) in the desired direction (usually the distance between objects, but this is not mandatory) to obtain a component of the force acting between the objects.

A. Available shapes

The goal of this model is to allow the user to easily compute the capillary force between two objects with parameters unavailable for usual axisymmetrical models. The choice was made to keep a reference axis z , similar to the symmetry axis of former models (see figure 2).

To develop complex shapes without having to define each point of the tip (or the object), analytical shapes have been used. In the z direction, usual profiles can be chosen : circular, conical or parabolical, while in the (x, y) plane, the section of the tip can be described as a polar function.

Elementary sections are

- circle : $r(\theta) = R$
- triangle : $r(\theta) = c/(2\sqrt{3} \cos \theta)$ for $-\pi/3 < \theta \leq \pi/3$
- square : $r(\theta) = c/(2 \cos \theta)$ for $-\pi/4 < \theta \leq \pi/4$

In a similar way, any regular polygon of side length c can be very easily implemented. Actually, virtually any section can be represented as it is developed in Fourier series (in polar coordinates) in order to obtain an analytical shape on the domain $\theta = [0; 2\pi]$.

The profile and section are then coupled to obtain the complete tip that has to be used in the equations. An example of a tip with parabolic profile and triangular section is shown on figure 2. The same formulation is also available for the object.

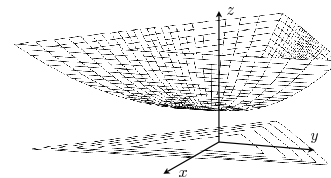


Fig. 2. Example of a tip and its projection on the (x, y) plane. Here, the section is a triangle and the profile is a parabola.

This model should now be soon applied to actual commercial tips, such as AFM tips.

V. THEORETICAL RESULTS

A. Introduction

The results presented in this section are twofold. First, validity of the code will be shown in section V-B. As literature only provides results for axisymmetrical shapes, those results will be the only ones that can be used as a proof for the model, since no result is available for other shapes.

Then, in section V-C, more complex shapes will be investigated.

B. Validation

Benchmark results used for comparison have to be separated into two groups :

- Exact solution : analytical solution [7], [16]
- Assumed shape meniscii [6], [9]

In [7], analytical solutions are derived from the base equations. They provide results for the sphere-plane and

sphere-sphere cases separated by a liquid bridge. The results (see figure 3) are presented with respect to the so-called filling angle ψ (see inner schematic of 3). One can see that the correspondance is very good with the model from [7] and that the approximation of [3] ($F = 4\pi R\gamma \cos \theta$) is valid for very small angles.

The discrepancies between the model results and [7] can be explained by the meshing of the meniscus. The mesh has to be limited to keep the computation time within acceptable boundaries. In general, the total computation time for a configuration is in the 1-5 minutes range .

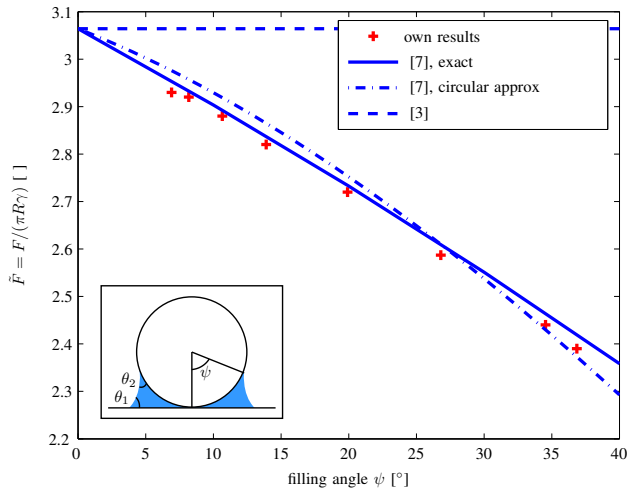


Fig. 3. Comparison of capillary force at the contact between a sphere and a plane for several models ($\theta_1 = \theta_2 = 40^\circ$). The positive value of force means an attractive force.

In [6], the shape of the tip is a parabola. The force values are in good agreement. We chose here to show the shape of the meniscus (see figure 4). It can be seen that –in the axisymmetrical case–, the circle approximation for the meniscus shape is valid under the considered conditions (see figure caption).

[9] and [16] also present the capillary force for different tip shapes (spheres, paraboloids). As the results of the model are as close with those literature results as for the other ones, they will not be reproduced here.

C. Tilted tips

In addition to the already existing results, our model is able to compute the capillary force for additional configurations, as detailed in section IV-A.

An important result which can be achieved with non axisymmetrical tips is the evolution of the capillary force with the tilt angle of the tip with respect to the object. Figure 5 shows the influence of the tilt angle τ on the force for a conical tip.

The conical tip for which the results are presented has an aperture angle α of 80° (the tilt angle is thus limited to a maximum of $90-80=10^\circ$). One can see that the force can vary from about 30 nN to over 400 nN, simply by tilting the tip over the plane.

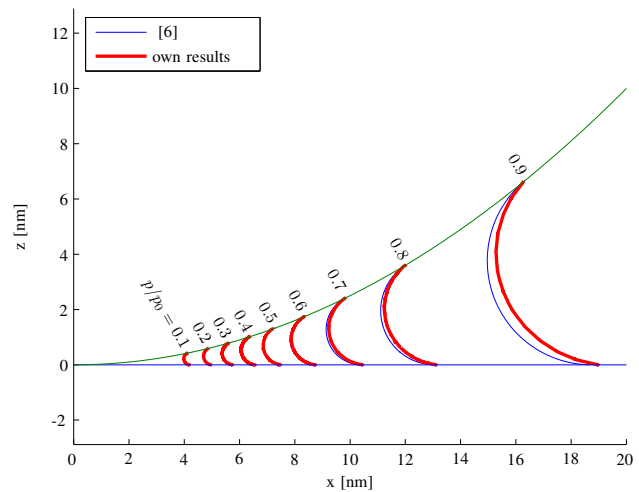


Fig. 4. Comparison of capillary menisci between a paraboloid (of curvature radius 20nm at the apex) and a plane for our model and the model from [6]. $\gamma = 72.5$ mN/m, $\theta_1 = \theta_2 = 0^\circ$

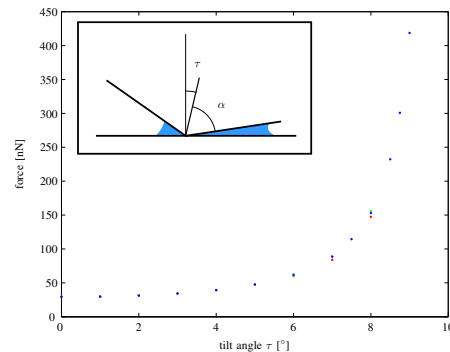


Fig. 5. Force between a tilted tip (the tilt angle is τ) and a plane. the aperture angle of the cone (α) is 80 degrees, temperature is 298 K the relative humidity is 90 %. Both contact angles are 30 degrees and the surface tension is 72 mN/m. The positive value of force means an attractive force. The different points at the same tilt value are for different mesh refinements. They give an idea of the numerical incertitude on the results.

VI. DISCUSSION

In the previous paragraphs, it was shown that our model could be used to compute the capillary force between two objects for usual shapes : spheres, cones, planes,... and reproduce existing results. In addition, the three dimensional capabilities of the model allow the user to compute capillary force for complex configurations with simple shapes or even with complex shapes (e.g. pyramids, rounded pyramids that can model the Berkovich AFM tips).

Possible applications

A very interesting result presented is that the force can easily be varied by a factor of more than 10. This ratio should allow the user to pick up and release a part by controlling the tilt angle of the tip with respect to the object.

To fix order of magnitude, the minimum and maximum forces are sufficient to lift respectively a cube of about $100 \mu\text{m}$ and $250 \mu\text{m}$ for a density of 2300 kg/m^3 (approximately the density of silicon). For objects with masses

between those values, a conical tip should be able to pick up, manipulate, and then release them in atmospheres of 90% of relative humidity.

The manipulable part weights can also be extended using different tip shapes or using multiple tips.

VII. EXPERIMENTAL VALIDATION

In order to validate the model, a test bed has been developed, following an AFM type design : a laser is reflected by an AFM tip onto a photodiode, using lenses and mirrors. If a force is applied on the AFM tip, the tip is deflected, the laser reflexion is modified and this modification can be measured using the photodiode. The laser spot displacement on the photodiode can then be converted to a force if the tip has a known stiffness.

The test bed is illustrated on figure 6. Currently, the force can be measured between an AFM tip and a substrate. The substrate can be moved over a 25 mm range with a 200 nm resolution, and over a further 200 μm range with a 1 nm resolution.

The tip and substrate are approached until contact, then the pull off is measured when retracting the substrate. As the tip and the substrate can be enclosed in a small box, humidity can be controlled and the variations of the force with respect to humidity can be measured. For validating purpose, a few pull offs were done using uncalibrated tips, as illustrated on figure 7.

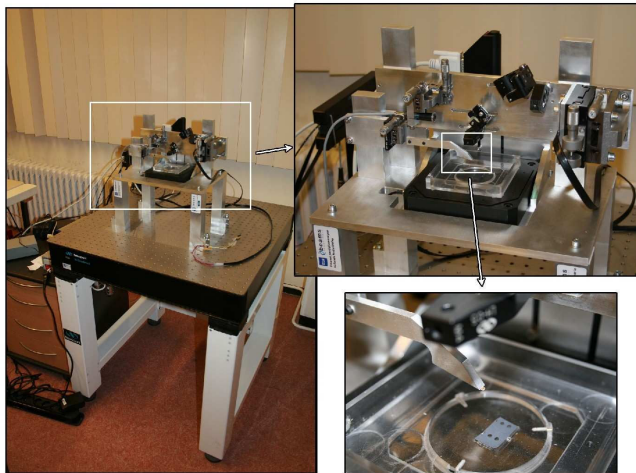


Fig. 6. Developed test bed : the left picture shows a general view of the test bed; the top right picture shows a closer view of the laser, lenses and mirrors and photodiode; the bottom right picture shows a close up view of the AFM tip holder.

VIII. CONCLUSIONS

This paper presents a three dimensional model for the computation of the capillary force. It allows to compute the effects of capillary condensation, with problems that are not mandatorily axisymmetrical. The model has been validated by comparing it with existing results. The new application shown here was the tilt angle of a conical tip with respect

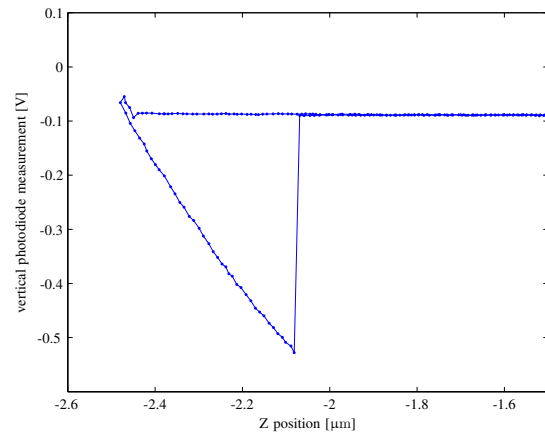


Fig. 7. Validation pull-off. The substrate is approached of the tip until contact (at the left). The substrate is then retracted until the photodiode signal takes its initial value. Our focus is on the pull-off, i.e. the vertical gap that expresses the value of the adhesion force. As it is validation pull-off, the AFM cantilver has not been calibrated and the y axis is only the photodiode measurement.

to a plane and the effects of this tilt angle on the capillary force between them.

The studied configuration shows that it should be possible to pick up and then release objects whose weight is between 50-500nN. The object could be picked up with a tilted tip, correctly positioned, and then released by reorienting either the tip or the object.

The next step will be to validate those numerical results through AFM experiments. For that purpose, a test bed was developed, which will be used to make force measurements.

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