Practical consideration on the identification of the kinematic parameters of the Stäubli TX90 robot

H. Hage*
MRI, 2PSR
EDF R&D
6 quai Watier , 78401, Chatou, France

P. Bidaud†
ISIR, CNRS UMR 7222
Université Pierre et Marie Curie
4 place Jussieu, 75005, Paris, France

N. Jardin‡
MRI, 2PSR
EDF R&D
6 quai Watier , 78401, Chatou, France

Abstract— *This paper describes a robust identification method of the kinematic parameters of robot manipulators, with minimal instrumentation, in order to demonstrate its validity in practical cases of use and especially in industrial environments. It is an autonomous, closed-loop and planar self calibration method. It is characterized by its efficiency, low cost and generic technique. It relies on the use of simple physical restraints applied to the effector. Practical ways to implement this method were carried out in EDF R&D laboratories on the Stäubli TX90 robot, a 6-axis industrial manipulator robot robot, in order demonstrate its convergence and effectiveness. The only data needed are the values of the joint positions of the robot, whose endpoints should reach 4 faces of a calibrated block. The reason for which this method is chosen is based primarily on the ease of its implementation, its speed and its accuracy. This study focuses on the identifiable parameters, the observability of the parameters, and the values of the kinematic parameters errors.*

Keywords: calibration, kinematic parameters, identifiable parameters, identification, robot manipulators

I. Introduction

The absolute accuracy of a robot is affected by different sources of disturbance and especially by the kinematic parameters errors whose nominal values, intervening in the kinematic models, were defined during the design of the robot. Despite the careful design of the robot, the nominal values are different from the real values, and the main errors are due to manufacturing tolerances and assembly. The solution is to establish a robot calibration which aims to identify the difference between nominal and real values.

Several methods have been proposed in the literature to calibrate the kinematic parameters of industrial robot manipulators. Most of them use a linearized model, and solve the system using least squares techniques. In this paper, it is shown that the use of sensors that measure the location of the terminal frame can be avoid. Instead, one can use simple physical restraints applied to the effector and still achieve fast, accurate and robust results, which makes it industrially interesting.

The calibration method described in this paper is based on the use of a touching probe that is very simple to use and on a calibrated block whose dimensions and properties are chosen optimally, in order not to cause a problem of observability. The reason for which this method is chosen is due to its simplicity, accuracy and its low cost. It certainly has some limitations and disadvantages, however, with about 95% accuracy improvement, it is among the best calibration methods and the best industrially.

The paper is organized as follows: the parameters defining the robot are presented in Section 2. Section 3 describes the calibration method. The experimental application is presented in Section 4. Section 5 describes the experimental results as well as the validation methods. Section 6 is the conclusion.

II. Kinematic description of the Stäubli TX90 robot

The TX90 robot is a serial manipulator robot with six rotational joints. A frame R_j is defined fixed on a link j (for $j = 0, \ldots, n = 6$), following the notations of the modified Denavit and Hartenberg method proposed by Khalil and Kleinfinger [1]. Two additional frames are defined: R_{-1} is the reference frame fixed relative to R_0 and R_7 is the end-effector frame fixed relative to R_6.

The transformation matrix ${}^{j-1}T_j$ from R_{j-1} to R_j can be obtained as a function of the following kinematic parameters: $\alpha_j, d_j, \theta_j, r_j, \beta_j$ [2], (for $j = 0, \ldots, n+1$), except for the first frame for which α_0, β_0, and d_0 are null. The joint angles are expressed by:

$$\theta = K \cdot \theta_c + \theta_{\text{offset}} \tag{1}$$

- K_j: the gains of joints transmissions assumed all equal to 1, because the values of joint angles θ_j are given directly in Stäubli controller system
- $\theta_{\text{offset}} = \left[0 \frac{-\pi}{2} \frac{\pi}{2} 000\right]^T$: represents the zero configuration of the TX90 robot (when $\theta_c = 0$)

The aim of this paper is to identify the errors of the kinematic parameters which are the difference between their real and nominal values. The initial number of parameters to identify is 37: $\Delta\alpha_{j;(j=1,\ldots,7)}$, $\Delta d_{j;(j=1,\ldots,7)}$, $\Delta\theta_{j;(j=0,\ldots,7)}$, $\Delta r_{j;(j=0,\ldots,7)}$, $\Delta\beta_{j;(j=1,\ldots,7)}$.

In the following tables, units are given in millimeters (mm) for the lengths and radians (rad) for the angles. The

*hiba.hage@edf.fr
†philippe.bidaud@upmc.fr
‡nicolas.jardin@edf.fr

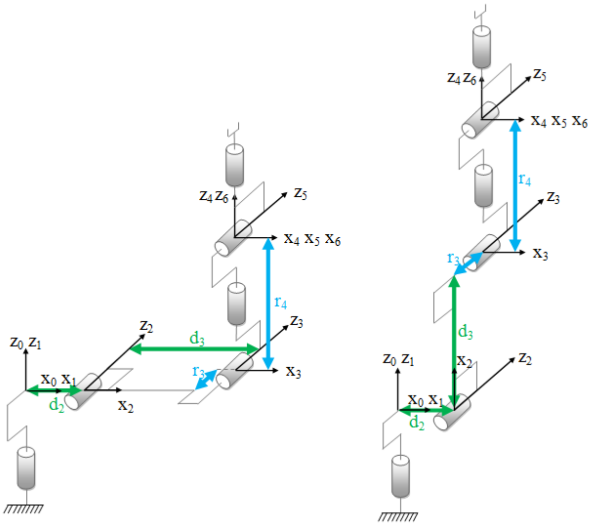

Fig. 1. The link frames and the D-H parameters of the TX90 robot.

j	α_j	d_j	θ_j	r_j	β_j	K_j
0	0	0	0	149.6	0	0
1	$\frac{\pi}{2}$	-625	θ_1	120.8	0	1
2	$-\frac{\pi}{2}$	50	θ_2	0	0	1
3	0	425	θ_3	50	0	1
4	$\frac{\pi}{2}$	0	θ_4	425	0	1
5	$-\frac{\pi}{2}$	0	θ_5	0	0	1
6	$\frac{\pi}{2}$	0	θ_6	0	0	1
7	0	0	-0.256	318.9	0	0

TABLE I. The kinematic Parameters of the TX90 robot

nominal values of the kinematic parameters of the TX90 robot are given in Table I. The link frames, the kinematic parameters of the TX90 robot, and the zero configurations before and after the addition of the θ_{offset} are shown in Fig. 1. The choice of ${}^{-1}T_0$ and ${}^{6}T_7$ will be explained in section 4.

III. Formulation of the calibration method

In this section, we present the calibration method, its formulation, and its resolution.

A. Calibration methods

A large variety of kinematic calibration methods of serial manipulators have been studied ([3], [4]). They are divided into two main groups based on the knowledge of the values of joint angles:

- **Open-loop calibration** based on the use of an external sensor to external measures such as measuring the location of the terminal frame ([5], [6]).
- **Closed-loop/autonomous calibration** based on a strain of at least one degree of freedom of the effector with a physical link with the outside environment ([7] - [11]).

A comparison study between the calibration methods of serial robots was done in [12].

Planar Calibration Method is a closed-loop calibration method based on using the values of the joint positions of a set of configurations of the robot whose endpoints are in the same plane. The most important advantages that make this method industrially interesting are: easy to implement, quick to apply, much cheaper than the methods with external sensor. Nevertheless, it has poverty of information, poor conditioning of the observation matrix, and needs high number of iterations to achieve the convergence, etc.

Several theories have been developed in this method to find solution to these drawbacks. The most important theory was to multiply the number of planes (3 to 4 planes). It was shown that if the robot has a prismatic joint, 3 planes will be sufficient ([8]- [9]). But if it consists of only revolute joints, one must apply the calibration using 4 planes, 3 of which are mutually orthogonal and the third chosen arbitrarily. An application of this method is carried out in [13] using a rectangular parallelepiped to get the 4 planes of contact. The chosen method in this paper is based on the use of a calibrated block to calibrate the TX90 robot.

B. The general equation of calibration methods

The kinematic calibration operation is very delicate: the numerical problem to be solved is nonlinear and the number of parameters is high (in the case of the TX90 robot it is equal to 37). The most robust procedures are those based on a linearized model that assumes low kinematic errors. The use of the linear model is much easier to carry, and it is the only way that can get accurate and robust results. In rare cases, when the resolution of the linearized model diverges or does not converge to an acceptable solution, the nonlinear optimization criterion in a least-squares method can be used. The linearized differential equation is presented in eq.(2).

$$\mathbf{\Delta y}\left(\mathbf{q},\mathbf{x},\xi\right) = \phi(\mathbf{q},\xi)\cdot\mathbf{\Delta}\xi + \rho \qquad (2)$$

- x: the cartesian variables of the situation of R_n
- q: the joint variables vector
- $\Delta\xi$: the vector of the errors between the vector of real values (unknown)ξ_r and the nominal values ξ of the kinematic parameters
- Δy: the output error between the model and the robot
- ϕ: (Jacobian matrix) $=\partial f/\partial\xi$(f is the general formulation of the equations of calibration which was linearized for the differential equation)
- ρ: the vector of observed errors due to modeling errors

To solve this system and estimate $\Delta\xi$, a set of a sufficient number of configurations $\mathrm{Q} = \{\mathrm{q}_1, \mathrm{q}_2, \ldots, \mathrm{q}_\mathrm{m}\}$ is applied in order to have an over-constrained system of equations.(eq.(3)).

$$\Delta Y\left(Q,X,\xi\right) = W(Q,\xi)\cdot\Delta\xi + \rho \qquad (3)$$

$$\Delta Y = \begin{cases} \Delta y^1(q^1, x^1, \xi^1) \\ \dots \\ \Delta y^m(q^m, x^m, \xi^m) \end{cases}$$

$$X = \{x_1, \dots, x_m\}$$

$$W = \begin{bmatrix} \Phi(q^1, \xi) \\ \dots \\ \Phi(q^m, \xi) \end{bmatrix} ; \begin{array}{l} W \in \mathbf{R}^{r \times N_p} \text{the observation matrix} \\ r \succ\succ N_p \end{array}$$

C. Formulation of the calibration method using 4 planes equations of a calibrated block

The general equation of a plane not passing through the origin is:

$$a'_i \cdot x + b'_i \cdot y + c'_i \cdot z + d'_i = 0 \quad (4)$$

By normalizing this equation, each point of coordinates (x, y, z) satisfies the standard equation of a plane:

$$a_i \cdot x + b_i \cdot y + c_i \cdot z + 1 = 0 \quad (5)$$

a_i, b_i, c_i are the coefficients of the plane i

The differential equation model shown in eq.(6) is calculated by using a first-order development while neglecting the second order terms and considering that the values of the coefficients are known without any error.

$$[a_i \cdot \psi_x(q, \xi) + b_i \cdot \psi_y(q, \xi) + c_i \cdot \psi_z(q, \xi)] \cdot \Delta\xi = -1 - \begin{bmatrix} P_x(q, \xi) & P_y(q, \xi) & P_z(q, \xi) \end{bmatrix} . \begin{bmatrix} a_i & b_i & c_i \end{bmatrix}^T \quad (6)$$

- ψ_x, ψ_y, and ψ_z are respectively the first three rows of the jacobian matrix defined in eq.(2). The calculation of the columns of the Jacobian matrix $(\psi\beta_i, \psi\alpha_i, \psi d_i, \psi\theta_i, \psi r_i)$ can be done as in [13]. They represent the displacement in translation and/or rotation of R_7 due to an error in the corresponding kinematic parameter.
- P_x, P_y and P_z are the cartesian coordinates of the end-point in the reference R_{-1}.

Applying eq.(6) for a large number of configurations of the same plane, we will have:

$$W(Q, \xi, a_i, b_i, c_i) \cdot \Delta\xi = \Delta Y(Q, \xi, a_i, b_i, c_i) \quad (7)$$

Applying eq.(7) on each of the 4 planes:

$$\begin{Bmatrix} W_1(Q_1, \xi, a_1, b_1, c_1) \\ W_2(Q_2, \xi, a_2, b_2, c_2) \\ W_3(Q_3, \xi, a_3, b_3, c_3) \\ W_4(Q_4, \xi, a_4, b_4, c_4) \end{Bmatrix} \Delta\xi = \begin{Bmatrix} \Delta Y_1(Q_1, \xi, a_1, b_1, c_1) \\ \Delta Y_2(Q_2, \xi, a_2, b_2, c_2) \\ \Delta Y_3(Q_3, \xi, a_3, b_3, c_3) \\ \Delta Y_4(Q_4, \xi, a_4, b_4, c_4) \end{Bmatrix} \quad (8)$$

Which is equivalent to:

$$W(Q, \xi) \cdot \Delta\xi = \Delta Y(Q, \xi) \quad (9)$$

$$W = \begin{bmatrix} W_1 & W_2 & W_3 & W_4 \end{bmatrix}^T$$

$$\Delta Y = \begin{bmatrix} \Delta Y_1 & \Delta Y_2 & \Delta Y_3 & \Delta Y_4 \end{bmatrix}^T$$

This brings back to the general calibration equation from which $\Delta\xi$ can be calculated.

D. Resolution of the calibration equation

If the rank b of W is maximum, a unique solution can be identified through the following relationship:

$$\hat{\Delta\xi} = W_w^+ \cdot Y ; W^+ = (W^T.W)^{-1}.W^T \quad (10)$$

W_w^+: the weighted pseudo-inverse of the observation matrix W. The reason for which the weighting is applied is explained at the end of this section.

Otherwise, it can be seen that some columns of W are dependent, and consequently this system of equations can be reduced so that the new observation matrix W_b contains only b independent columns chosen arbitrarily [14]. $\Delta\xi_b$ combine the errors of the parameters that correspond to these columns and called identifiable parameters:

$$\Delta Y(Q, X_t, \xi) = W_b(Q, \xi) \cdot \Delta\xi_b \quad (11)$$

It is absolutely necessary to go through a step of determination of the identifiable parameters before computing the errors on kinematic parameters. First, the zero columns of W and the corresponding parameters should be eliminated because they have no effect on the model. And then a QR decomposition of the matrix W should be applied as follows:

$$W = Q. \begin{bmatrix} R & 0 \end{bmatrix}^T \quad (12)$$

- $W \in \mathbf{R}^{r \times c}$ the observation matrix
- $Q \in \mathbf{R}^{r \times r}$ an orthogonal matrix
- $R \in \mathbf{R}^{c \times c}$ an upper triangular matrix

Theoretically, the non-identifiable parameters correspond to the elements of the diagonal of the matrix R whose absolute value is less than a tolerance τ_{QR}:

$$\tau_{QR} = \text{r} \cdot \text{Epsilon} \cdot \max |R_{ii}| \quad (13)$$

And consequently, the number of identifiable parameters can be calculated as follows:

$$b = \sum_i |R_{ii}| \succ \tau \quad (14)$$

Several methods can be used to calculate $\Delta\xi_b$ including the least squares method that seeks to minimize the difference ΔY between the model and the robot. It is based on calculating the weighted pseudo-inverse of the observation matrix as shown in eq.(15):

$$\Delta\xi_b = W_{b,w}^+ \cdot \Delta Y \quad (15)$$

The equation will be solved to get the least squares error solution to the current parameter estimate. Successive iterations should be carried out until the error $\Delta\xi_b$ becomes sufficiently small. After each iteration, the kinematic parameters should be updated in W_b and ΔY. The condition number of W_b is considered as the most accurate and sensitive index of the observability of the parameters in the calibration system of equations:

$$\text{cond}(\text{W}_b) = ||W_b|| \cdot ||W_b^+|| \quad (16)$$

To find it, simply divide the largest singular value to the smallest $(\sigma_{\max}/\sigma_{\min})$. Ideally, the condition number must be close to one. Note that the value of the condition number

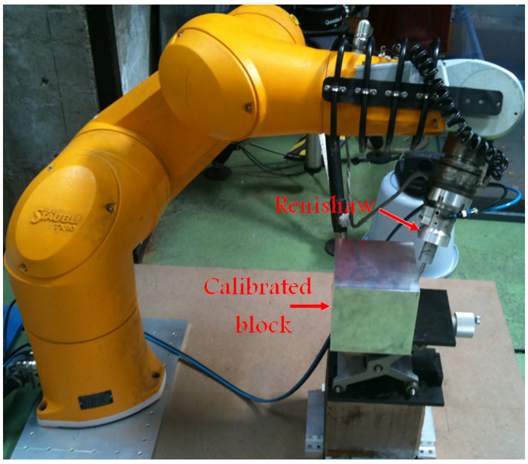

Fig. 2. Calibration Test of the TX90 robot.

and consequently, the final accuracy depends on the unit of the kinematic parameters. In fact, the kinematic parameters are of different natures (lengths and angles) and the calculation of $\Delta\xi_b$ is based on inversing the observation matrix W_b whose elements depends on the units of the kinematic parameters. In order to get accurate results and to prevent the condition number to tend towards very high values, it is so important to choose wisely the unit of the kinematic parameters. In section 5, it is shown that the use of meter for lengths and radians for angles can get the best results. To avoid making a choice of units, one can apply the weighted pseudo-inverse of the observation matrix ($W_{b,w}$) in order to get dimensionless columns without affecting the errors values.

IV. Experimental application

In order to carry out this method and have an over-constrained system of equations, the robot should reach a high number of configurations per plane, and consequently the largest possible number of parameters among the 37 kinematic parameters could be identified. The chosen number of configurations per plane is 37. The only data needed, which is the values of the joint positions of the robot, are collected from the industrial Stäubli control system (version CS8). In order to achieve successfully the calibration, one must respect the following descriptions of equipments and recommendations. Note that the trials were conducted in laboratories and with equipments of EDF R&D.

A. *The choice of the calibrated block*

In order to have three adjacent planes that are mutually and perfectly orthogonal and a fourth plane also perfectly parallel to the first plane, a calibrated block is used so that four of its faces are used for the calibration contacts. The choice of the block properties is very important and must satisfy the following needs:

1. The dimensions must be practical and should not pose a problem of reachability. In other terms, the robot should be able to reach easily the largest possible area of each of the 4 faces of the bloc and to change its configurations while touching the different samples on the block. The optimal choice of the dimensions (for the TX90 calibration) is: 150x150x150 (mm^3), with 0.02 mm dimension accuracy.
2. The choice of the configurations on each face is also of great importance. Because of choosing a low-dimensional block, the configurations should be well distributed over the entire face of each of the 4 planes of the block so that it does not create an observability problem.
3. The calibration properties of the block should be carefully chosen, otherwise, the results will not be satisfactory and reliable. The calibration properties are: perpendicularity tolerance of 0.1 mm/m, flatness tolerance of 0.04 mm/m and surface roughness Ra of 0.8 μm (very smooth).
4. It should be cheap while respecting the properties mentioned previously, therefore, aluminum is chosen.

B. *The choice of the sensor tool*

Each of the four faces of the block will be touched with about 37 different configurations and well distributed over the entire surface. The control of the contact between the effector and the plane will be carried out using the LP2 Renishaw touching probe shown in Fig. 2. It will be mounted on the tool changer. It is a dynamic triggering probe that consists of a body and a stylus with a ball connected to a LED that lights up whenever there is a contact. This sensor is used because it is very accurate, efficient, and fast measuring while being so simple to use and not expensive which is also industrially important. It has a sense of directions along x, y and z and does not have a degree of freedom, and therefore, R_7 is considered fixed relative to R_6.

$$
{}^6T_7 = \begin{bmatrix} cos(-14.7^\circ) & -sin(-14.7^\circ) & 0 & 0 \\ sin(-14.7^\circ) & cos(-14.7^\circ) & 0 & 0 \\ 0 & 0 & 1 & 318.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

C. *The block position relative to the robot*

The quality of the results varies from one position to another, therefore, the choice of the block position relative to the robot is also very important. The optimal position of the block, so that the robot reach the maximum surface of the 4 faces, is shown in Fig. 3. Multiple choices of the reference frame R_{-1} are possible. The normal to the faces define the axes and the origin O_{-1} is located in the center of the bottom face. In this way, the frame R_{-1} will be at equal distance from all the faces of the block in order to eliminate the possibility of additional errors. Its coordinates (in mm) relative to the frame R_0 are shown in Fig. 3. The coefficients of the 4 planes of the calibrated bloc, in R_{-1}, according to the equation eq.(5) are shown in Table II.

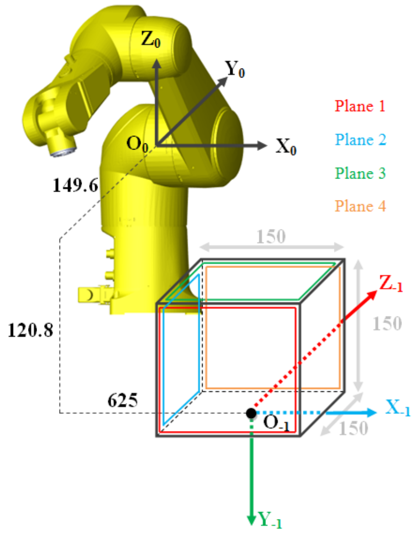

Fig. 3. The axes of the reference frame R_{-1} on the block.

Plane j	a_j	b_j	c_j
Plane 1	0	0	13.33
Plane 2	13.33	0	0
Plane 3	0	6.67	0
Plane 4	0	0	−13.33

TABLE II. The coefficients of the 4 planes in R_{-1}

V. Results and validations

A. *Experimental trials*

Several tests were made in order to reach the best results. The quality of the results was affected by the following elements:

- the number of configurations per plane
- the tolerance of the QR decomposition τ_{QR}
- the unit of the kinematic parameters if the weighting is not carried

The elements that judges the quality of the result are:

- The number of identifiable parameters which ideally should be as large as possible.
- The condition number of the observation matrix which ideally should be as small as possible (and especially in calibration case it should be in the order of 10^3 or less).

Test results are noted below:

1. At first, the 37 configurations per plane were used while using the tolerance τ_{QR} as defined in (13). The use of meter as the unit for the lengths (if the weighting is not carried) leads us to the best results among all units (convergence of the iterations, lower condition number). As a result, the number of identifiable parameters was 25 with a condition number in the order of 10^4 which is not a satisfactory result.
2. To optimize the results, several criteria were changed (unity, tolerance, etc.). The value of τ_{QR} was increased, in order to eliminate the parameter(s) whose singular value is very low and has a bad influence on the results. Finally, with $\tau_{QR} = 10^{-1}$ and meter as unit, a condition number of the order of 10^3 was reached with 24 identifiable parameters. The results were not acceptable because the error of the r_3 parameter was slightly high in comparison to its nominal value (r_3=50 mm, $\Delta r_3 \approx 8$ mm).
3. Even by reducing the number of configurations to 27 per plane, and so the remaining ten configurations can be used to validate the results, the same problem was found: the only value that was not reasonable was Δr_3. If the parameter α_4 is not considered among the parameters to identify, the number of identifiable parameters is 23 and Δr_3 is now reasonable. The parameter α_4 was chosen, because it can have an influence on r_3 and it is the parameter that has the lowest singular value among the 24 identified parameters.
4. As a result, instead of not taking into account α_4, the fourth test was in reducing the number of configurations per plane to 25 with $\tau_{QR} = 10^{-1}$. The number of identifiable parameter is 23. The evolution of the condition number of the observation matrix during the different iterations is shown in Fig. 4. Convergence is reached after 4 iterations with 7.5×10^3 as the condition number of the observation matrix which is a satisfactory result as a start.

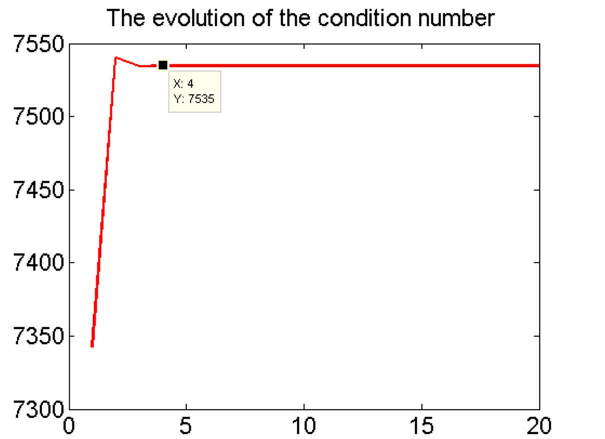

Fig. 4. The evolution of the condition number of the observation matrix.

The kinematic parameters errors are shown in Table III. One can notice that the number of identifiable parameters and the value of the condition number of the observation matrix are related to many parameters including: the number of configurations selected per plane, the value of tolerance of the QR decomposition set, the unit of kinematic parameters, etc. In the case of the TX90 robot, the maximum number of identifiable parameters, but not necessarily the best value of the condition number of the observation matrix was obtained by choosing more than 30 configurations per plane, with the value of tolerance that was set by definition. The best result was obtained with 25 configurations per plane, a tolerance of 0.1. Choosing such tolerance, the parameters whose singular values are the lowest and have a negative effect on the result of calibration can be eliminated.

In Tables III, IV, V:

- '-' represents the parameters that are not considered among the parameters to identify
- '$\times$' represents the parameters that have no effect on the model
- '/' represents the parameters whose effect is grouped in other parameters

B. Validation procedures

In this section, different practical methods of validation are presented to prove that this calibration method is accurate and precise. The errors are added to the nominal values of the kinematic parameters and therefore, they are considered as the new initial parameters.

B.1 Validation using the same set of configurations

New calibration iterations are conducted by using the same set of 25 configurations per plane and the new initial parameters. The new identified kinematic errors are very low (*10^{-11}) as shown in Table IV.

B.2 Validation using a different set of configurations

Calibration iterations are conducted by using the set of 10 different configurations per plane (well distributed over the surfaces), and the new initial parameters. The new identified kinematic errors are as shown in Table V. The values of the identified errors are very low in comparison to the nominal values and the errors initially identified, which proves that the calibration method is robust whatever the points used for the calibration and provide a precision discussed later.

B.3 Validation by comparing nominal and calibrated distances to their measured values by using the same set of points

Validations can be done in the cartesian space by using the tool in possession which is the probe. Since the distance between O_{-1} and each face of the block have been already measured (with a precision of 0.02 mm), the distance between O_{-1} and the configurations of each of the 4 planes can be calculated numerically along the axis perpendicular to each face. Consequently, a comparison between the the mean of the theoretical/measured values to the numerical/calculated values can be established.

The kinematic model of the 25 configurations per plane is calculated, with respect to R_{-1}. At first the nominal parameters and then the calibrated parameters are used as shown in Fig. 5–8. The vector that defines the position of R_7 which is the end of the probe, with respect to R_{-1} is: ${}^{-1}P_7 = [Px, Py, Pz]^T$. The comparison between the measured values to the mean of the calculated values is shown in Table VI. The calibrated values of the distances are quite close to the measured values. The rate of improvement between the nominal and the calibrated distances with respect to the measured distances is $\succ 94\%$. Note that the measurement errors (sensor, position and dimensions of the block) are very low compared to the errors on the kinematic parameters. This validates the results.

j	$\Delta\alpha_j$	Δd_j	$\Delta\theta_j$	Δr_j	$\Delta\beta_j$
0	$-$	$-$	0.0050	-8.3836	$-$
1	-0.0046	-0.8920	0.0083	-0.9588	/
2	-0.0012	0.9559	-0.0017	/	/
3	0.0004	3.2636	0.0044	0.6675	0.0006
4	/	-2.2895	0.0027	3.1939	/
5	/	-1.1318	-0.0036	-0.4481	/
6	/	/	/	/	/
7	0.0027	-1.3547	$\times$	3.7510	/

TABLE III. The kinematic Parameters errors of the TX90 robot

B.4 Validation by comparing nominal and calibrated distances to their measured values by using a different set of points set of points

The kinematic model of 10 different configurations per plane is calculated, with respect to R_{-1}, by first using the nominal parameters and then the calibrated parameters (Fig. 9–12). The comparison between the measured values to the mean of the calculated values is shown in Table VII. The calibrated values of the distances are very close to the measured values, and the rate of improvement between the nominal and the calibrated distances is $\succ 94\%$.

j	$\Delta\alpha_j^{e^{-11}}$	$\Delta d_j^{e^{-11}}$	$\Delta\theta_j^{e^{-11}}$	$\Delta r_j^{e^{-11}}$	$\Delta\beta_j^{e^{-11}}$
0	$-$	$-$	0.00	-0.06	$-$
1	0.00	0.46	-0.00	-0.18	/
2	-0.00	-0.39	0.00	/	/
3	0.00	-0.62	-0.00	-0.12	/
4	/	0.60	-0.00	-0.53	/
5	/	0.31	-0.00	0.14	/
6	/	/	/	/	/
7	/	-0.02	$\times$	-0.57	/

TABLE IV. The errors after validation using the same set of points

j	$\Delta\alpha_j$	Δd_j	$\Delta\theta_j$	Δr_j	$\Delta\beta_j$
0	$-$	$-$	0.0005	-0.1552	$-$
1	/	0.2523	-0.0001	-0.0448	/
2	/	0.7211	-0.0006	/	/
3	/	0.0629	0.0004	0.1697	/
4	/	0.3444	-0.0008	-0.0595	/
5	/	0.3444	-0.0015	0.1492	/
6	/	/	/	/	/
7	/	/	$\times$	/	/

TABLE V. The errors after validation using a different set of points

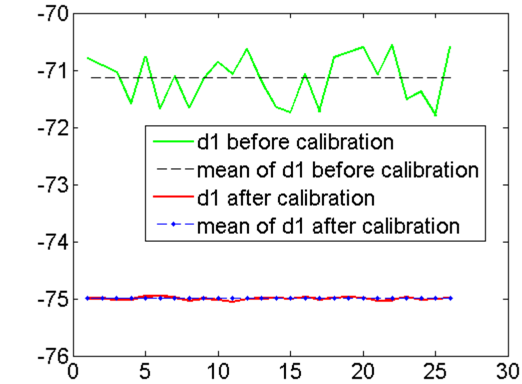

Fig. 5. The nominal and calibrated d_1 using the 25 points.

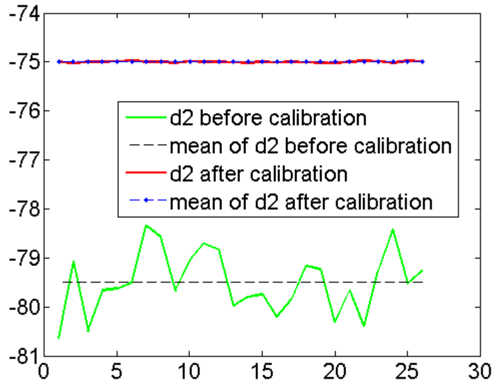

Fig. 6. The nominal and calibrated d_2 using the 25 points.

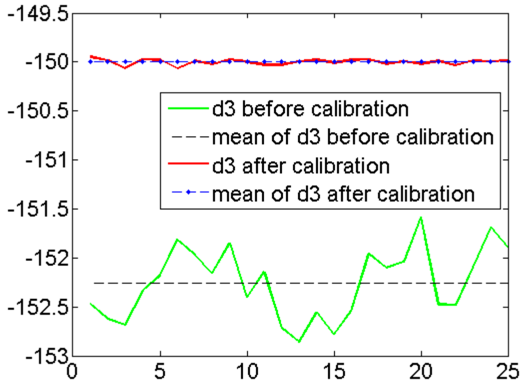

Fig. 7. The nominal and calibrated d_3 using the 25 points.

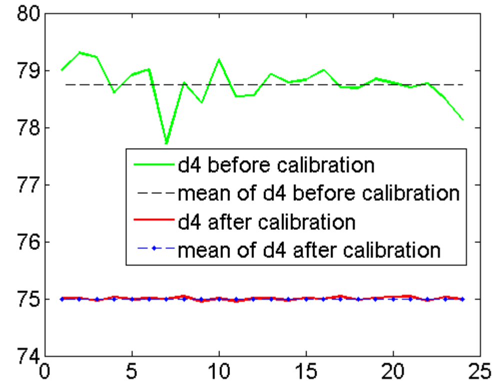

Fig. 8. The nominal and calibrated d_4 using the 25 points.

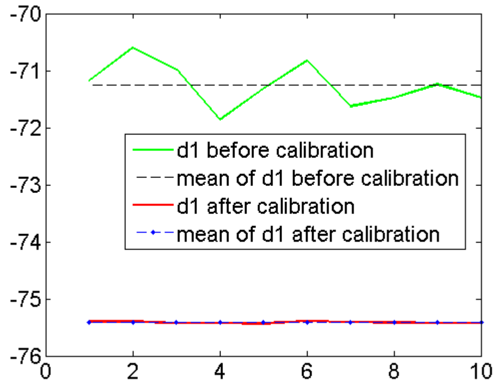

Fig. 9. The nominal and calibrated d_1 using the 10 points.

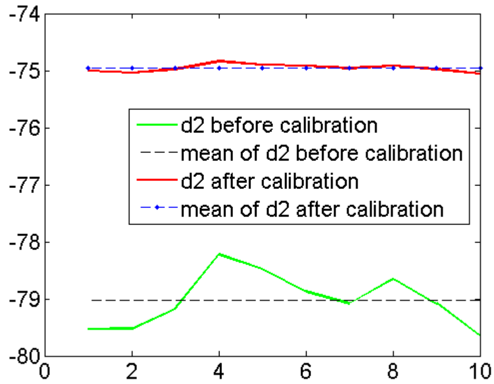

Fig. 10. The nominal and calibrated d_2 using the 10 points.

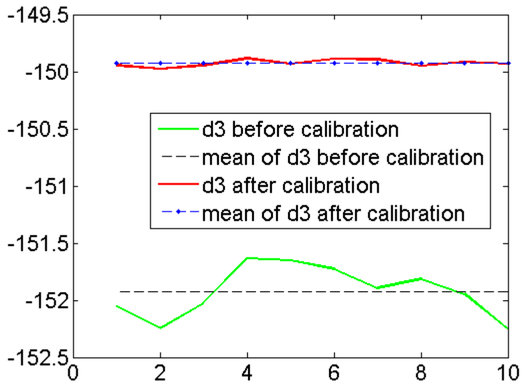

Fig. 11. The nominal and calibrated d_3 using the 10 points.

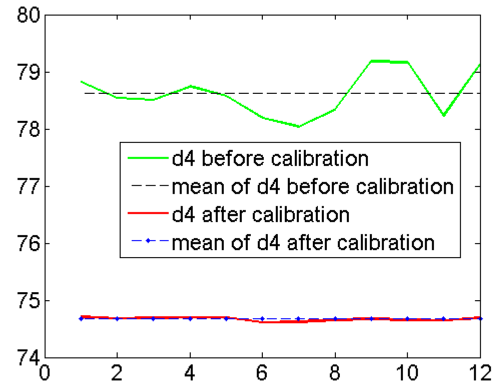

Fig. 12. The nominal and calibrated d_4 using the 10 points.

d_j	**measured**	**nominal**	**calibrated**	**%**
$d_1 = P_z$	-75	-71.14	-74.99	94.85
$d_2 = P_x$	-75	-79.5	-75.00	94.00
$d_3 = P_y$	-150	-152.3	-150.00	98.46
$d_4 = P_z$	75	78.75	74.99	95.00

TABLE VI. Comparison between the measured, nominal and the calibrated distances for the 25 configurations/plane

d_j	**measured**	**nominal**	**calibrated**	**%**
$d_1 = P_z$	-75	-71.26	-75.41	94.45
$d_2 = P_x$	-75	-79.03	-74.95	94.57
$d_3 = P_y$	-150	-151.9	-149.92	98.68
$d_4 = P_z$	75	78.62	74.66	94.72

TABLE VII. Comparison between the measured, nominal and the calibrated distances for the 10 configurations/plane

VI. Conclusion

This paper presents a study of an autonomous calibration method using 4 planes equations of a calibrated block. The method is easy and quick to implement and its ability is proved in industrial environments. It was carried out on the Stäubli TX90 robot in EDF R&D laboratories. It is a low cost method, robust and able to get a precise knowledge of the kinematic parameters values. The estimation of the identifiable kinematic parameters is carried out using QR decomposition and weighted iterative-pseudo inverse technique. It is fast to converge (3 to 5 iterations) and the number of identifiable parameters is high (23 to 25). Nevertheless, one should remember that the kinematic calibration does not give a perfect knowledge of the position of the end effector (rate of knowledge improvement is about 94 %) that is why the effect of deformation must be taken into account afterwards.

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