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Robust obstacle crossing of a wheel-legged mobile robot using minimax force distribution and self-reconfiguration

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***Abstract*— This paper focuses on the problem of robust obstacles crossing for a high mobility wheel-legged robot. To improve the obstacle clearance capability, a method dealing with the contact stability optimization is developed. A specific stability criterion taking the friction into account is proposed. The optimization algorithm uses both the kinematic redundancy in order to modify the position of the Center of Mass (CoM), modifying the resulting distribution of contact forces, and the actuation redundancy to improve the stability of frictional contacts by adapting the internal forces. We show that the choice of this particular criterion allows us to maximize the robustness of contacts stability relatively to the modeling errors affecting force control (friction in mechanical transmission). Performances of this algorithm are evaluated in simulation and the necessity for a CoM trajectory planning is highlighted by an analysis of obstacle crossing using this criterion.**

I. INTRODUCTION

Wheel-legged hybrid mobile robots are systems designed to increase mobility on uneven terrains and obstacles crossing capacities. Systems like those proposed in [1] and [2] are hybrid locomotion systems consisting in polyarticulated arms playing the role of active suspensions ending by wheels producing the traction and the steering. They have two interesting characteristics for control purposes. Firstly, as legged robots, hybrid locomotion systems have the ability to move their center of mass (CoM) with respect to the contact points with the ground. This may allow the robot to reach a more stable posture during its evolution. Secondly, they have an actuation redundancy which, combined with their active suspensions, allow to modify the force distribution in order to optimize a particular criterion. This problem is often called the force distribution problem and has been widely studied for legged or hybrid systems, grasping and co-manipulation.

Basically, two problems have to be addressed simultaneously for the mobility of robots over rough terrains: slippage handling and tip-over avoidance. Uneven terrain and obstacles present steep slope, making the traction forces needed to cross them difficult to produce without loosing adhesion. Furthermore, the complex geometry encountered can cause tumbles, endangering the integrity of the system.

The problem addressed in this paper is the one of frontal obstacle crossing by wheel-legged mobile robots. We assume that the motion is slow enough for the inertial effects to be neglected. At a given time, we compute the contact geometry based on the robot sensors informations, and search for the most stable force distribution and center of mass (CoM) location. A new force distribution is then calculated considering the actual robot configuration and a task force computed by a PID controller in order to follow the desired CoM position. While doing this, a torque is produced by the wheels for propulsion on the obstacle.

Various methods have been used to specify the position of the robot CoM. Most of them are designed to avoid tip over [3]. They led to several stability criterion based on the CoM projection on the support base created by the contact points or based on others characteristic points such as in [4] (the Zero Moment Point) to insure stability. Finally [5] compute torques needed to overturn the robot around the support base's axes. Such methods are widely used in legged robot locomotion, however they don't consider slippage between the wheels and the ground.

Criteria designed for slippage avoidance are mostly used for the selection of appropriate grasp configurations. A survey of these measures can be found in [6]. Most of them are based on the smallest wrench the system can sustain without violating contact constraints [7].

The force distribution problem has been solved by considering several different criteria. Some minimize the ratio between the normal contact force and the tangential contact force at each contact [8], [9]. Others use a trade-off between energy consumption and contact stability using linear matrix inequalities [10].

Fig. 1. The HyLoS2

Contact stability is very sensitive to modeling errors like friction in mechanical transmissions affecting the contact force control. We propose here a control method for obstacle crossing maximizing robustness relatively to these perturbations. It exploits force distribution optimization and self-reconfiguration in order to maximize a criterion based on stability robustness, considering at the same time, tip-over and slippage. An analysis tool for crossing prediction is then derived from this method. The relevance of the criterion introduced here is discussed and its definition is given in section II. Section III presents formulation of the optimization problem and describes the algorithm used to solve it. Finally, section IV gives simulation results for the wheel-legged robot HyLoS2 (Fig. 1) and presents a numerical analysis of an obstacle cross-ability.

II. CONTACT STABILITY

A. Previous work

Criteria proposed in the literature have different objectives, decreasing energy consumption, increasing available traction, or improving stability. In this paper, we consider obstacle crossing as a local need, and we will not take energy consumption into account.

A frequently used criterion is the largest ratio between the i^{th} tangential force and the i^{th} normal force over the set of contacts. An optimization of this measure tends to maximize the traction capabilities while ensuring contact stability if it is achievable. Furthermore, it seems to be the best solution to ensure stability when the friction coefficient and the contact normal are badly estimated. This is due to the fact that this optimization will minimize the angle between the contact force and the estimated normal. However, it can result in very weak forces on some contact, allowing the mobile robot to be close to tip-over or slippage, even more if this type of criterion is used to choose the CoM position.

Works on grasping often use a measure which express the combination of constraints in the wrench space associated with the CoM of the grasped object. As a result, this gives all the wrenches that are resistible if applied on the CoM. Different criteria derive from this constraint representation. [12] uses the smallest wrench applicable on the CoM as a criterion to choose a grasp configuration. [13] solves the force distribution problem by maximizing the residual ball radius in this space, making sure that the robot will be able to add forces in order to support the largest possible disturbance. Here, the only disturbances the robot's CoM will undergo are the inertial forces and the robot's orientation estimation errors resulting from them. Thus, it is not necessary to consider disturbances in every direction. [14] consider only the direction the disturbances are suspected to act on. If the chosen direction is the one of the inertial effect caused by the obstacle crossing, this method will result in the best solution for traction purposes by allowing the largest acceleration to be produced. However, as this method does not consider individually the contact forces, the stability of some contacts may be reduced for the sake of global stability.

Criteria based on linear matrix inequalities (LMI) lead to control algorithm where the radius of the residual ball inscribed in the constraints of each contact are somehow evened, but these methods are designed for the force distribution problem and are not easy to formulate for the posture control problem.

B. Proposed stability criterion

Contact constraints may be defined by Coulomb's friction law where, to insure stability, each contact force must lie in the coulomb friction cone defined by:

$$\begin{array}{rcl} \mu^2 f_{z_i}^2 & < & f_{x_i}^2 + f_{y_i}^2 \\ f_{z_i} & > & 0 \end{array} \tag{1}$$

where μ is the static friction coefficient at the contact, $\mathbf{f_i} = \begin{bmatrix} f_{x_i} & f_{y_i} & f_{z_i} \end{bmatrix}^T$ is the contact force vector at the i^{th} contact point, f_{x_i}, f_{y_i} and f_{z_i} being the components of the force along the contact frame's axis $\Re_{c_i}$, z_i is the contact normal, x_i the longitudinal direction and y_i the lateral direction. [11] uses a conservative simplification of these constraints by seeing them as a pyramid instead of a cone:

$$\begin{array}{rcl} \frac{\mu_i}{\sqrt{2}} f_{iz} + f_{ix} & > & 0 \\ \frac{\mu_i}{\sqrt{2}} f_{iz} - f_{ix} & > & 0 \\ \frac{\mu_i}{\sqrt{2}} f_{iz} + f_{iy} & > & 0 \\ \frac{\mu_i}{\sqrt{2}} f_{iz} - f_{iy} & > & 0 \end{array} \tag{2}$$

Let us define the vector $\mathbf{d}$ as:

$$\mathbf{d} = A\ \mathbf{f} \tag{3}$$

where A is a matrix defined by $A = blockdiag(A_i)$, A_i being the constraint matrix from (2):

$$A_i = \begin{bmatrix} 1 & 0 & \frac{\mu_i}{\sqrt{2}} \\ -1 & 0 & \frac{\mu_i}{\sqrt{2}} \\ 0 & 1 & \frac{\mu_i}{\sqrt{2}} \\ 0 & -1 & \frac{\mu_i}{\sqrt{2}} \end{bmatrix}$$

and $\mathbf{f} = \begin{bmatrix} \mathbf{f_1}^T & \dots & \mathbf{f_n}^T \end{bmatrix}^T$ is a vector containing all contact forces. The elements of $\mathbf{d}$ represent the tangential force that can be added to the contact force of a leg in each tangential direction without breaking the contact stability (Fig. 2).

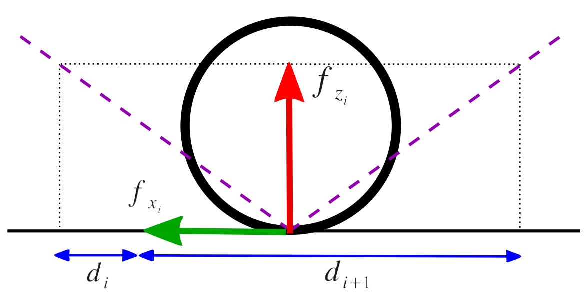

Fig. 2. Residual forces on a contact

By maximizing the smallest element of $\mathbf{d}$, we maximize the perturbations that can be sustained by the robot. The proposed optimization criterion is then:

$$\phi(\mathbf{f}) = \min(A\ \mathbf{f}) \tag{4}$$

where $\min(\mathbf{x})$ is the function returning the smallest element of the vector $\mathbf{x}$. If $\phi(\mathbf{f})$ is positive, then every constraint of (2) is respected and all the contacts are stable. On the other hand, if it is negative, then at least one contact is sliding or broken.

III. Optimization problem formulation

The optimization problem must be formulated in such a way that it allows to determine the CoM position and the contact forces efficiently from a computational point of view. This section first introduce the force distribution equations of the system, and presents the force distribution problem along with its compact form. We then show how the CoM location can be explicitly added to the problem and propose an algorithm to solve non linear minimax optimization

A. Force distribution equations

We consider a locomotion system made of n legs (Fig. 3). The i^{th} leg is in frictional contact with the ground through the i^{th} wheel at point P_i. The coordinates of this point in the frame $\Re_p$ (fixed for the moving platform S_p with its origin located on the CoM considered fixed) are grouped in $\mathbf{p_i}$.

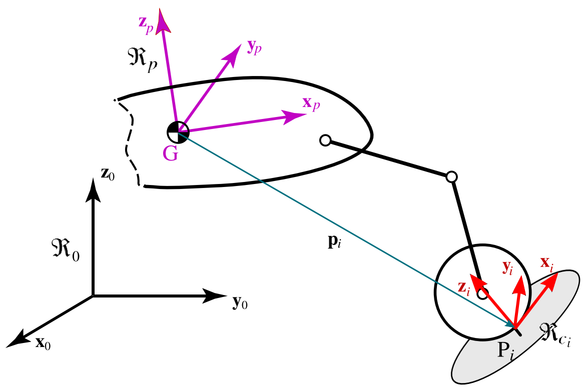

Fig. 3. Model of a polyarticulated system

The equations describing the equilibrium for a system with mass concentrated in the moving platform are given in a matrix form by :

$$G\ \mathbf{f} = \mathbf{F} \tag{5}$$

where $\mathbf{F}$ is the set of the external and inertial wrench applied on S_p and G is a $[6 \times 3n]$ matrix giving the equivalent wrench to the contact forces in $\Re_p$.

$$\mathbf{G} = \begin{bmatrix} R_{c_1} & \dots & R_{c_n} \\ \tilde{\mathbf{p}}_1\ R_{c_1} & \dots & \tilde{\mathbf{p}}_n\ R_{c_n} \end{bmatrix}$$

where $\tilde{\mathbf{p}}_i$ is the skew symmetric matrix of the cross-product associated to the vector $\mathbf{p_i}$ and R_{c_i} is the rotation matrix from $\Re_p$ to $\Re_{c_i}$.

In addition, the contact forces must satisfy some additional constraints:

- actuator limits
$$\begin{array}{rcl} J^T\ \mathbf{f} & < & \tau_{max} \\ -J^T\ \mathbf{f} & < & \tau_{max} \end{array} \tag{6}$$
where $J = blockdiag(J_i)$, J_i being the Jacobian matrix of the i^{th} leg and τ_{max} is the torque limit vector.
- contact constraints previously expressed in (2).

The system (5) has six equations for $3n$ unknowns. If the locomotion system has 3 legs or more, the problem is sub-definite, making an optimization process possible. This process will aim at maximizing our criterion.

B. Force distribution problem

The force distribution problem may be expressed as:

$$\begin{array}{ll} \text{Find } \mathbf{f} \in \mathbb{R}^{3n} \text{ maximizing} & \phi(\mathbf{f}) \\ \text{Subject to:} & \\ & \begin{array}{rcl} G\mathbf{f} & = & \mathbf{F} \\ J^T\ \mathbf{f} & < & \tau_{max} \\ -J^T\ \mathbf{f} & < & \tau_{max} \end{array} \end{array} \tag{7}$$

In order to reduce the constraint dimension of this problem, we transform it into its compact primal form as described in [15].

The solution f can be separated in a particular solution $\mathbf{f_p}$ and an homogeneous solution $\mathbf{f_h}$.

$$\mathbf{f} = \mathbf{f_p} + \mathbf{f_h} \tag{8}$$

The particular solution is given by the weighted pseudo-inverse

$$\mathbf{f_p} = G^+_{Norm}\ \mathbf{F} \tag{9}$$

The homogeneous solution, often called the internal forces, is such as:

$$G\ \mathbf{f_h} = \mathbf{0} \tag{10}$$

It belongs to the null space of G described by:

$$\mathrm{Ker}(G) = \mathrm{vect}(\{\mathbf{b_i}\}_{i \in [1,m]}) \tag{11}$$

$\{\mathbf{b_i}\}_{i\in[1,m]}$ being the vectors of the basis of $\mathrm{Ker}(G)$, with $m = dim(G) - rank(G)$.

If we define $N_f = [\mathbf{b_1}\ \dots\ \mathbf{b_m}]$, we can write the homogeneous solution as:

$$\mathbf{f_h} = N_f\ \mathbf{x_f} \tag{12}$$

where $\mathbf{x_f} \in \mathbb{R}^m$ becomes the problem unknown.

With this variable change, the criterion (4) becomes:

$$\phi(\mathbf{x_f}) = \min(A\ G^+\mathbf{F} + A\ N_f\ \mathbf{x_f}) \tag{13}$$

and the constraints (6) are:

$$\begin{array}{c} J^T\ N_f\mathbf{x_f} < \tau_{max} - J^T\ G^+\ \mathbf{F} \\ -J^T\ N_f\mathbf{x_f} < \tau_{max} + J^T\ G^+\ \mathbf{F} \end{array} \tag{14}$$

Problem (7) can now be rewritten as:

$$\begin{array}{ll} \text{Find } \mathbf{x_f} \in \mathbb{R}^m \text{ maximizing} & \min\left(A[G^+\mathbf{F} + N_f\ \mathbf{x_f}]\right) \\ \text{Subject to:} & \\ & J_f^T\ \mathbf{x_f} < \mathbf{a_f} \end{array} \tag{15}$$

with

$$J_f = \left[\begin{array}{cc} N_f^T \ J & -N_f^T \ J \end{array} \right]^T$$

and

$$\mathbf{a_f} = \left[\begin{array}{cc} (\tau_{max} - J^T \ G^+ \ \mathbf{F})^T & (\tau_{max} + J^T \ G^+ \ \mathbf{F})^T \end{array} \right]^T$$

C. Optimization of the CoM position

If we define $\mathbf{x_p} = [X \quad Z]^T$ as the vector containing the longitudinal and vertical coordinates of the CoM position in an intermediate frame chosen with a vertical z-axis and a x-axis being in the direction of the projection of x_p on the plane perpendicular to z. The null space of G is entirely defined by the contact geometry, so for fixed contact points, the null space of G is not modified by a change of X or/and Z. Furthermore for a frontal crossing, *i.e.* the rotation between the ground and the contact frames are around y_p, the particular solution $\mathbf{f_p}$ can be stated as a combination of X and Z:

$$\mathbf{f_p} = \mathbf{c_x} \ X + \mathbf{c_z} \ Z + \mathbf{f_0} \tag{16}$$

Thus, the CoM position can be added to the optimization variable by expressing the contact forces as:

$$\mathbf{f} = \mathbf{f_0} + N \ \mathbf{x} \tag{17}$$

with $N = \left[\begin{array}{ccc} N_f & \mathbf{c_x} & \mathbf{c_z} \end{array} \right]$ and $\mathbf{x} = \left[\begin{array}{cc} \mathbf{x_f}^T & \mathbf{x_p}^T \end{array} \right]^T$

The vector $\mathbf{x_p}$ must lie within the geometrical capabilities of the robot, *i.e*. the vectors $\mathbf{p_i}$ must not have a length larger than the maximum length of a leg noted D_{max}.

The problem is now written as:

$$\begin{array}{ll} \text{Find } \mathbf{x} \in \mathbb{R}^{m+2} \text{ maximizing:} & \min\left(A\left[\mathbf{f_0} + N \ \mathbf{x}\right]\right) \\ \text{Subject to} & \\ & J_p^T \ \mathbf{x} < \mathbf{a} \\ & \|\mathbf{p_i}\| < D_{max} \end{array} \tag{18}$$

with

$$J_p = \left[\begin{array}{cc} N^T \ J & -N^T \ J \end{array} \right]^T$$

and

$$\mathbf{a} = \left[\begin{array}{cc} (\tau_{max} - J^T \ \mathbf{f_0})^T & (\tau_{max} + J^T \ \mathbf{f_0})^T \end{array} \right]^T$$

It must be noticed that conversely to the problem (15), the inequality constraints in (18) are not linear. This is due to the fact that the Jacobian matrix of the legs are modified by the CoM position.

D. Dutta's Solving algorithm

(15) and (18) are called minimax problem. We present briefly the solving algorithm for constrained minimax problem described by [16]. This algorithm is designed for the following kind of problem:

$$\begin{array}{lrcl} \text{Minimize} & F(\mathbf{x}) & = & max \ f_i(\mathbf{x}) \\ \text{Subject to} & & & \\ & g_j(\mathbf{x}) & < & 0 \\ & h_l(\mathbf{x}) & = & 0 \end{array} \tag{19}$$

with $i \in I = \{1 \ ... \ n\}$, $j \in J$ and $l \in L$, f_i, g_j and h_l are continuously differentiable functions of $\mathbf{x}$

Let denote $\overline{\mathbf{x}}$ the optimal solution of the problem. The algorithm is as follow

- step 1. Choose an optimistic guess ϕ_o of the optimal value of F (*i.e.* $\phi_o \leq F(\overline{\mathbf{x}})$). Set $k = 0$
- step 2. Minimize

  $$P(\mathbf{x}, \phi_k) = \sum_{i \in I(\mathbf{x})} [f_i(\mathbf{x}) - \phi_k]^2 + \sum_{j \in J(\mathbf{x})} w_j g_j^2(\mathbf{x}) + \sum_{l \in L} v_l h_l^2(\mathbf{x})$$

  with

  $$I(\mathbf{x}) = \{i \in I : f_i(\mathbf{x}) > \phi_k\}$$

  $$J(\mathbf{x}) = \{j \in J : g_j(\mathbf{x}) > 0\}$$

  and w_j, v_l are pre-specified weights.
  Denote the solution of this minimization as $\overline{\mathbf{x}}_k$.
- step 3. Set $\phi_{k+1} = \phi_k + \sqrt{P(\overline{\mathbf{x}}_k, \phi_k)/n}$.
- step 4. If $\phi_{k+1} - \phi_k < \varepsilon$ where ε is a prespecified small number, stop the algorithm. Otherwise, set $k = k+1$ and go to step 2.

[16] gives proof that under mild assumption, ϕ_k converges to $F(\overline{\mathbf{x}})$, so $\overline{\mathbf{x}}_k$ converges to a solution of the problem (the solution if only one exists).

IV. NUMERICAL ANALYSIS

In this section, the simulation results are presented. The control algorithm is evaluated on the HyLoS2 robot using the open source dynamic simulator Bullet .

A. Preamble

The Hylos2 is an actively actuated polyarticulated system consisting of 4 legs supporting the chassis. Each leg is composed of 2 segments and a wheel with four actuated degrees of freedom (dof): two moving the segments in the saggital plane, one for the steering and one for propulsion (Fig. 4).

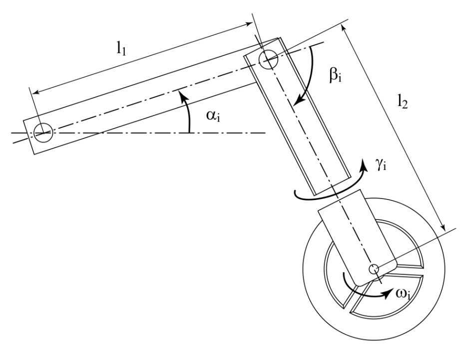

Fig. 4. A leg of the HyLoS II robot

For simplification of the control law:

- We consider only frontal obstacle crossing, thus the lateral forces don't need to be taken into account. This has two consequences on the model used:
  - The second line of the matrix G is suppressed

– The simplification of a pyramidal friction as described in (2) is not necessary:

$$A_i = \begin{bmatrix} \mu_i & 1 \\ \mu_i & -1 \end{bmatrix}$$

- The robot speed is sufficiently low to neglect the inertial effects on the CoM. The task is only to compensate for gravity ($\mathbf{F} = \begin{bmatrix} 0 & -Mg & 0 & 0 & 0 \end{bmatrix}^T$) and the displacement of the robot is created by an additional torque on the wheel. As the vertical height does not affect the gravitational force distribution, it is eliminated from the optimization, the only position parameter which is optimized is X.

For computational purposes, the optimal position is calculated every 100 time step of the simulation (1ms) by solving (18). Given a new desired configuration, the position of the CoM is controlled by adding a force to the task vector $\mathbf{F}$ in the force distribution problem (15). This force is computed by a PID controller. The force distribution optimization (15) is performed every 10 time step using the simulated CoM position (not the desired one) and the task force vector modified by the PID controller. The null space's basis N_f is given by a singular value decomposition of the matrix G and step 1 of Dutta's algorithm is done by setting ϕ_o to 10000 which, considering the weight of the robot and its torque limitations, is way above the maximum achievable stability.

The distance between two wheels on the same side is fixed and the static friction coefficient is set to 0.8.

B. Results

The robot is asked to climb a step with a height representing 25% of the distance between the wheels (Fig. 5). The stability margin defined in (4) is plotted on Fig. 6 and the simulated and desired horizontal position are given Fig. 7.

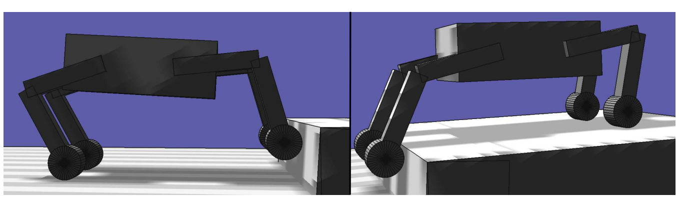

Fig. 5. Pahes 3 of the step crossing

During phase 1, the robot is on a horizontal flat ground. The front wheels touch the step and it enters in phase 2, the stability margin drops below zero, indicating that it cannot produce the necessary forces to start the climbing motion. The desired position is calculated and the robot reaches the optimal configuration. During this phase, the robot still uses the ground to sustain its weight. Once the stability margin has increased enough, the robot starts to climb over the step and it enter in phase 3. While climbing, the optimal position of the CoM is updated, avoiding possible tip-over as the robot is rising. Once the front wheels have reached the top of the step, the robot is once again in a phase similar to phase 1 where all the wheels are on horizontal planes. And the same phases follows: the rear wheels touch the obstacle, the robot

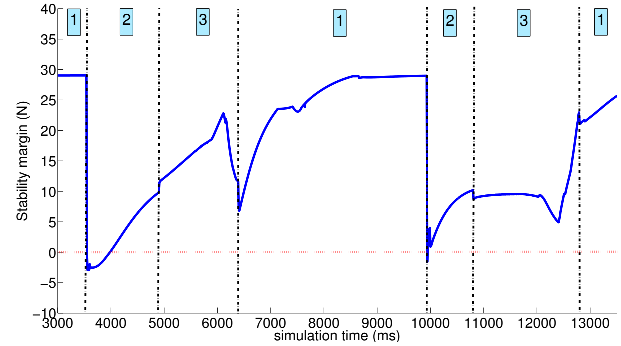

Fig. 6. Stability margin during a step crossing

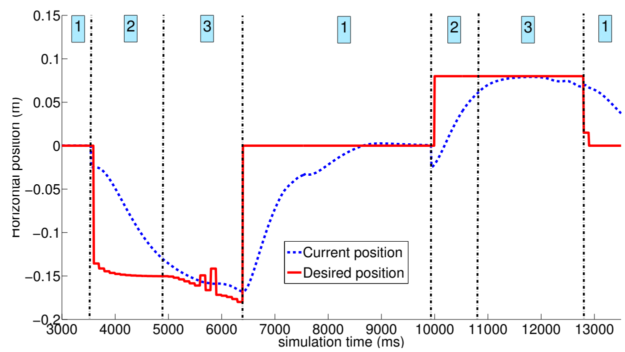

Fig. 7. Desired and actual horizontal position of the CoM

reaches the optimal configuration and the wheels climb over the step.

To study the robustness of the algorithm, we run another simulation, but this time we add a white noise of 10% on the motor torque. The measured stability margin is plotted in Fig 8. It must be noted that as this is the measured margin, it does not drop below zero during phase 2 because the robot is using the ground instead of the obstacle to sustain itself.

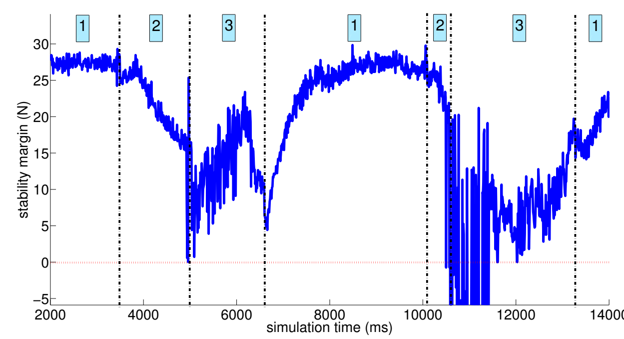

Fig. 8. Stability margin during a step crossing with reduced efficiency

The robot manages to go through the first three phases of the climbing process. However when the rear wheels try to climb the step, the stability drop below zero, the robot slip several times before climbing. This result was predictable considering Fig. 6. The stability margin for the earlier phases

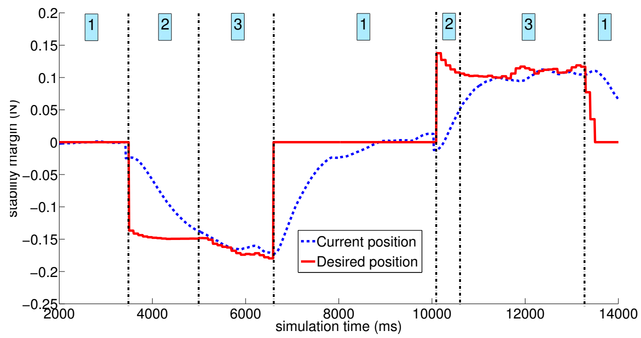

Fig. 9. Desired and actual horizontal position of the CoM with reduced efficiency

was higher than the margin for the latter phases, indicating a more robust climbing for the front wheels than for the rear wheels.

The reactive algorithm presented above has some limits. During phase 2, the stability margin drops below zero, however thanks to the double contact of the wheel (with the step and with the ground), stability is kept. If the robot were asked to climb the step down, there would not be a double contact and the robot would fall. Furthermore, discontinuities in the desired position can imply great forces created by the position control. It might be necessary to analyze the obstacle before crossing it.

C. Obstacle analysis

To analyze a known obstacle, we can solve problem (15) for each geometrical configuration the robot will have to manage during the crossing and for each possible positions of the CoM. Fig. 10 show the map giving the stability margin depending on the chosen CoM position and on the progress of the crossing for an obstacle composed of a step to climb up followed by a step to climb down. For comprehension of the figure, unstable and impossible configuration are lowered and the optimal position is drawn in white.

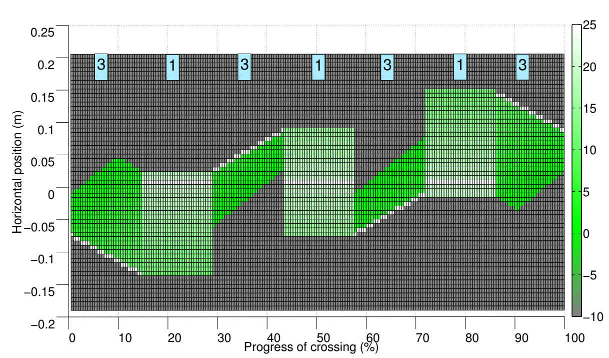

Fig. 10. Map of the stability margin depending on the robot's situation

This figure gives prominence to the discontinuity of the optimal CoM position. Furthermore, the optimal position for a geometrical configuration of the contacts can be an unstable one for the next configuration (at 57% of the crossing). Thus, a trajectory planning of the CoM allowing a smooth, quick and safe crossing is needed. The idea is to use a map like Fig. 10 to find a continuous trajectory of the CoM position. If such a trajectory does not exist, then the crossing might be dangerous or even impossible.

V. Conclusion

A criterion designed in order to indicate the robustness of a mobile robot contacts stability has been proposed. It is based on the smallest perturbation sustainable at the contact level. A control algorithm computing both the best force distribution and the best configuration of the robot relative to this criterion is described. Simulations show good results of the control scheme for a step crossing, however, analysis of an obstacle crossing using the criterion shows that a trajectory planning of the CoM is necessary for a safe crossing.

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