

Understanding Parent-Infant Behaviors Using Non-negative Matrix Factorization

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Abstract. There are considerable differences among infants in the quality of interaction with their parents. These differences depend especially on the infants development which affects the parent behaviors. In our study we are interested on 3 groups of infants: typical development infants (TD), autistic infants (AD) and mental retardation infants (MR). In order to identify the groups of signs/behaviors that differentiate the development of the three groups of children we investigated a clustering method *NMF* (non-negative matrix factorization), an algorithm based on decomposition by parts that can reduce the dimension of interaction signs to a few number of interaction behaviors groups. Coupled with a statistical data representation *tf-idf*, usually used for document clustering and adapted to our work, *NMF* provides an efficient method for identification of distinct interaction groups. Forty-two infants and their parents were observed in this study. Parent-infant interactions were videotaped by one of the parents at home during the first two years age of child.

Keywords: Parent-infant interaction, Clustering, *tf-idf*, *NMF*, Autism.

1 Introduction

Parent-infant interaction has an important contribution to infant development, particularly during the first two years age of child. However, there are some developments and psychological pathologies that can affect the parent-infant interaction; one of these pervasive developmental disorders is autism.

Autism is a severe psychiatric syndrome characterized by the presence of abnormalities in reciprocal social interactions, abnormal patterns of communication, and a restricted, stereotyped, repetitive repertoire of behaviors, interests, and activities. Although it is a well-defined clinical syndrome after the second, and especially after the third, years of life, information on autism in the first two years of life is still lacking [1]. Home movies (movies recorded by parents during the first years of life, before diagnosis) and direct observation of infants at high

risk due to having an autistic sibling are the two most important sources of information for overcoming this problem. They have both described children with autism disorder (AD) during the first year of life (and also in the first part of the second year) as not displaying the rigid patterns described in older children [2]. In particular, from a clinical point of view, the autistic children can gaze at people, turn toward voices and express interests in communication as typically developing infants do. It is of seminal importance to have more insight into these social competencies and in which situations they preferentially emerge in infants who are developing autism. However, there are various signs that differentiate children with ASD from children with developmental delays. The autistic child characterized by less of a response to their name, less looking at others, lower eye contact quality and quantity, less positive facial expression and intersubjective behaviors (e.g., showing shared attention).

In our study, we focus on automatic characterization of the interaction quality of the different group of children by age with the goal to study further specific early signs in the interactive field. We studied home movies of the first 18 months of life of 42 infants categorized in 3 groups. Then, using a statistical representation of data and an automatic clustering technique, we performed a longitudinal analysis of the first 3 semesters and found significant results. These results are confirmed by the clinicians analysis.

The general view of the study is presented in Fig. 1. After data collection, all parent and infant behaviors are annotated. After that, an interaction database has been created by extracting all interaction sequences, parent behaviors and infant behaviors co-occurring, in a window of 3 seconds. Then, quantitative statistics have been performed to describe the interaction, and to assess emergence of language and social engagement by time and by group. Then we investigated a statistical method usually used for document analysis, *tf-idf*, to make a statistical representation of interaction data. This statistical characterization presents an input of the clustering algorithm.

This paper is organized as follows. Section 2 presents the longitudinal corpus and the annotation process. Section 3 presents the multimodal interaction database. Section 4 describes our method for statistical data representation and clustering. Experimental results of the proposed method are provided in section 5. In a last section, some concluding remarks and the direction of future works are presented.

2 Home Movies Corpus and Annotation

2.1 Corpus

The database used in our study contains 3 groups of children matched for gender and socio-economic status, with family home movies (HM) running for a minimum of 10 minutes for each of the 3 first semesters of life.

We selected 42 infants and their parents from the Pisa home movies database [3], with the following criteria: 15 children (10 male and 5 female) who will be diagnosed with autism disorder (AD), 12 children (7 male and 5 female) with

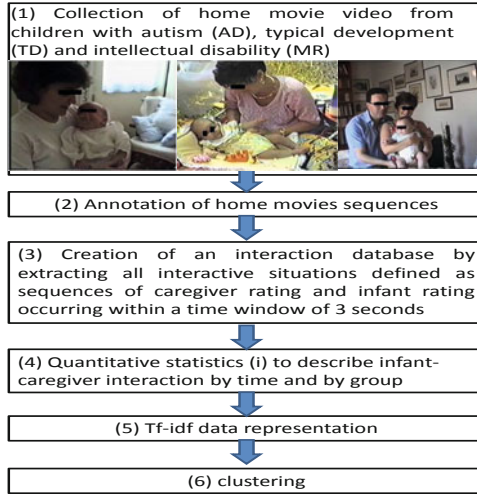


Fig. 1. Interaction analysis

Table 1. Number and duration (in min) of different scenes from the three semesters analyzed in the home movies database of the three group of children

	AD (15)	MR (12)	TD (15)	<i>t-test</i> (<i>p</i> value)		
				AD vs MR	AD vs TD	MR vs TD
Number <i>Semester 1</i>	162	100	164	.116	.163	.138
Duration <i>Semester 1</i>	258.5	148.5	237.3	.142	.759	.105
Number <i>Semester 2</i>	164	111	201	.639	.088	.079
Duration <i>Semester 2</i>	258.1	176.3	347.5	.863	.148	.278
Number <i>Semester 3</i>	127	90	120	.917	.747	.701
Duration <i>Semester 3</i>	212	194.5	194.5	.202	.474	.371

intellectual disability, mental retardation (MR), and 15 (9 male and 6 female) with a history of typical development (TD). All scenes showing a situation in which an interaction can occur (i.e. all scenes with an infant and an adult) were extracted. Table 1 shows the total number of scenes and their duration. Preliminary *t-test* analysis was used to verify that chosen video material was comparable across groups and for each range of age, in length and number of standard situations (table 1).

2.2 Annotation

After data collection, the annotation process is a fundamental task. The annotators used a computer-based coding system called Observer which is a complete manual event recorder, designed for collection, management and analysis of observational data. It was configured for the application of the Infant Caregiver

Table 2. Parent Behaviors

Behaviors	E/S	Description
Regulation up/down	State	The caregiver modulates the child's arousal and mood. The caregiver may act to either excite or calm-down the child.
Caregiver's solicitation vocalising/naming/gesturing/ touching/showing object/request	Event	Within a sequence of interaction, the caregiver stimulates the child requesting attention by vocalizing, naming, gesturing, touching him/her or showing him object

Behavior Scale to our video media file-material [4]. 4 coders were trained to use Observer 4.0, until they achieve a satisfactory agreement (Cohen's Kappa $\geq 0,7$) with an expert clinician in almost 6 matched sequences. The standard situations derived from the home movies of the 3 groups of children (AD, MR and TD) were mixed and each one was rated by one trained coder (blind to the group belonging). 25% of standard situations were randomized and rated by 2 coders independently, with a final inter-rater reliability, calculated directly by the Observer program, showing a satisfactory Cohen-mean value (0.75 to 0.77).

Table 2 and 3 are composed successively of items referring to the ability of the infant to engage in interactions and to caregiver's actions towards the infant. Behaviors are grouped into 4 independent classes (Table 4) created to allow the coders to focalize their attention selectively on one class:

- Infant Basic Behavior: composed of 6 simple items regarding social engagement.
- Infant Complex Behavior: composed of 10 items regarding a more complex level of social engagement (they can sometime overlap the previous, basic, level of description).
- Infant Vocalization: composed of 3 items regarding vocal activity towards the other.
- Caregiver Behavior: composed of 6 items describing caregiver's solicitation or stimulation toward the infant, to obtain his attention.

All target behaviors (table 4) can be described as Events which take an instant of time and only frequency of occurrence (rate) matters for an Event. Some of the behaviors can also be described as states which take a period of time, have a distinct start and an end; rate and duration matter for a State.

The clinical study [5] confirms that babies who have become autistic, particularly during the first six months of life, can display intersubjective behaviors; nevertheless, the duration of these behaviors, such as enjoying with people and maintaining social engagement, are reduced in infants with autism and the duration of syntony emerges as an item which significantly differentiates infant with autism from typical infants and mentally retarded infants.

3 Multi-modal Interaction

We first created a multimodal interaction database by extracting all interactive situations defined as sequences of parents rating and infant rating occurring

Table 3. Infant Behaviors

Behaviors	E/S	Discription
Looking at people/at object/around	State	The child directs his/her eyes towards an object, or a human face, or simply looks around.
Orienting toward people/object/name prompt	Event	The child assumes a spontaneous gaze direction towards a new sensory stimulation coming from a people/object or towards the person.
Gaze Following a person/an object/a gaze of a person	Event	The child shifts his gaze to follow the gaze or the trajectory of another person, or object.
Smiling at people/at object	Event	The child intentionally smiles at a person or at object.
Seeking contact with person/with object	Event	The child employs spontaneous and intentional movements to reach contact with a person or with an object.
Explorative activity with person/object	State	The child touches something to find out what it feels like. The exploring activity may be done by hands, mouth or other sensory-motor actions.
Enjoying with person/with object	State	The child finds pleasure and satisfaction experiencing a physical and/or visual contact with a person or with an object.
Sintony	State	The child shows a sintonic response to the other's mood; he/she shows signs of congruous expressions to affective environmental solicitations.
Anticipation of other's intention	Event	The child makes anticipatory movements predicting the other's action.
Communicative gestures	Event	The child displays use of social gestures.
Referential gaze	Event	The child shifts his/her gaze towards the caregiver to look for consultation in a specific situation.
Soliciting	Event	The child displays a verbal, vocal, or tactile action to attract the partner's attention or to elicit an other kind of response.
Accept Invitation	Event	The child's behavior is attuned to the other person's solicitation within 3 seconds from the start of stimulation.
Offering him/her self	Event	The child offers parts of his/her body to the other person.
Imitation	Event	The child repeats, after a short delay, another person's action.
Pointing	Event	The child shifts his gaze towards the direction indicated by parent uses his/her finger to indicate something in order to share an emotional experience
Maintaining social engagement	State	The child takes up an active role within a two-way interaction. The child interacts, vocalises and maintains turn taking.
Simple Vocalisation	Event	The child produces sounds towards people or objects.
Meaningful Vocalisation	Event	The child intentionally produces sounds with a stable semantic meaning
Crying	State	The child starts crying after a specific/non specific event.

Table 4. Parent and Infant Behaviors tags

Infant Basic Behaviors	tags	Infant Complex Behaviors	tags	infant Vocalizations	tags	Parent solicitation	tags
Looking People	B_lkp	ENJOYING*	C_ejx	simple	v_sim	REGULATION*,UP	G_reu
Looking object	B_lko	ENJOYING*,WITH PEOPLE	C_ejp	meaningful	v_mea	REGULATION*,DOWN	G_red
Looking around	B_lka	ENJOYING*,WITH OBJECT	C_ejo	CRYING	V_ery	request behavior	g_req
Explorative Act/Object	B_exo	SINTONY	C_snt			naming	g_nam
Contact People	b_ctp	SOCIAL ENGAGE	C_seg			touching	g_tou
Contact Object	b_cfo	anticipation	c_ant			vocalising	g_voc
Orienting XXX	b_orx	comm. gestures	c_com			gesturing	g_ges
Orienting People	b_orp	referential gaze	c_ref			show object	g_sho
Orienting Object	b_oro	soliciting	c_sol			NULL (cg)	G_nul
Orienting to Name	b_orn	acc invitation	c_acc				
Gaze Follow Person	b_gfp	offering	c_off				
Gaze Follow Object	b_gfo	imitation	c_imi				
Smiling People	b_smp	pointing	c_poi				
Smiling Object	b_smo						

within a time window of 3 seconds. Moreover, all infant states (with a duration) that demonstrate synchrony, reciprocity, quality of interaction (called social behaviors): enjoying with people or object, syntony, social engagement were automatically integrated in the interaction database. Then, we performed quantitative statistics to describe the interaction, and to assess emergence of language and social engagement by time and by group.

After creation of the multimodal interaction database, we obtained $m = 176$ different interaction behaviors, Fig. 2, Fig. 3 and Fig. 4 show the 20 most frequent

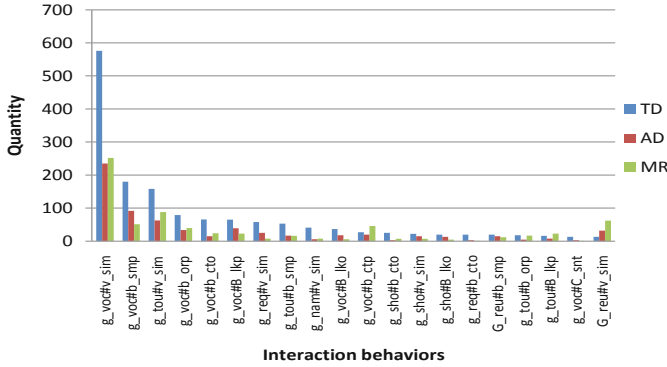


Fig. 2. 20 most frequent interaction behaviors in the first semester

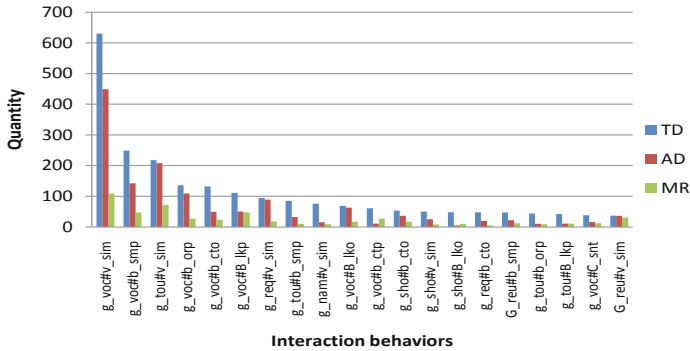


Fig. 3. 20 most frequent interaction behaviors in the second semester

interaction behaviors successively in the first, second and third semesters by group of infant (G_reu#V_sim means that regulation up, G_reu, produced by parent and the response of the child by a simple vocalization, V_sim, co-occurring in a time window of 3 seconds).

However, this quantitative representation of interaction behaviors is not always informative for grouping and distinguishing the different behaviors dyads. Therefore, we propose to compute a statistical characterization of data based on *tf-idf* (term frequency-inverse document frequency) representation.

4 Parent-infant Interaction Analysis

After data collection, data characterization process is the key problem for parent-infant interaction analysis to allow the automatic processing of this data. In this context, this section describes the usually adopted statistical representation method as *tf-idf* (term frequency-inverse document frequency), then the clustering method in order to extract the different groups of interaction behaviors.

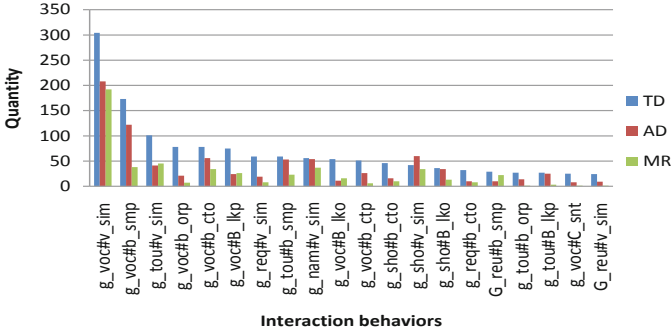


Fig. 4. 20 most frequent interaction behaviors in the third semester

4.1 Statistical Interaction Characterization

The first step in social signals analysis is to transform the scenes annotations into a representation suitable for the learning algorithm and the clustering task. This leads to an attribute value representation of interaction. A simple way to transform the annotations into a feature vector is using a statistical *tf-idf* representation which is widely used in information retrieval applications, where each feature is a single interaction behavior.

The *tf-idf* weight (term frequency-inverse document frequency) is evolved from IDF which is proposed by Sparck Jones [6] with the heuristic intuition that a query term which occurs in many documents is not a good discriminator, and should be given less weight than one which occurs in few documents. This weight is a statistical measure used to evaluate how important a word is to a document in a collection or corpus. The importance increases proportionally to the number of times a word appears in the document but is offset by the frequency of the word in the corpus. Variations of the *tf-idf* weighting scheme are often used by search engines as a central tool in scoring and ranking a document’s relevance given a user query. One of the simplest ranking function is computed by summing the *tf-idf* for each query term; many more sophisticated ranking functions are variants of this simple model. Similar to the *tf-idf* normalization steps for document word matrices, a *tf-idf* procedure is applied to represent statistically the parent-infant interaction. We adapt this method for our interaction model problem. First, we simply count the number of times a given behavior appears by movie. Then, this count is usually normalized to prevent a bias towards longer movies (which may have a higher term count regardless of the actual importance of that term in the movie) to give a measure of the importance of the term t_i within the particular movie d_j . Thus we have the term frequency, defined as follows.

$$tf_{ij} = \frac{n_{ij}}{\sum_k n_{kj}} \tag{1}$$

where n_{ij} is the number of occurrences of the considered dyad of behavior (t_i) in the movie d_j , and the denominator is the sum of number of occurrences of all the dyads in the movie d_j .

The inverse document frequency is a measure of the general importance of the term (obtained by dividing the total number of movies by the number of movies containing the interaction behavior, and then taking the logarithm of that quotient).

$$idf_i = \log \frac{|D|}{|\{d : t_i \in d\}|} \quad (2)$$

where $|D|$ is the total number of movies in the database and $|\{d : t_i \in d\}|$ is the number of movies where the interaction behavior t_i appears (that is $n_{ij} \neq 0$). Then, we calculate:

$$(tf - idf)_{ij} = tf_{ij} \times idf_i \quad (3)$$

4.2 Clustering

Non-Negative Matrix Factorization. Non-negative matrix factorization (*NMF*) has been studied under the name of positive matrix factorization [7] at the early stage. It is then popularized by the work of Lee and Seung and has been found lots of applications in text mining [8] [9]. Recent advancement of *NMF* has shown that it shares much similarity with K-means and spectral clustering methods, and is capable of producing good cluster capability [10]. However, *NMF* is more difficult algorithmically because of the non-negativity requirement but provides a more intuitive decomposition of the data.

Given a non-negative matrix X in size $n \times m$, *NMF* factorizes it into two non-negative matrices W and H (Fig. 5),

$$X = WH \quad (4)$$

where W is a $n \times k$ matrix and H is $k \times m$, while k is usually much smaller than both n and m . Define the loss as the square of the Euclidean distance between X and the reconstructed matrix $\bar{X} = WH$,

$$\min_{W \geq 0, H \geq 0} \|X - \bar{X}\|^2 = \sum_{i=1}^n \sum_{j=1}^m (X_{ij} - \bar{X}_{ij}) \quad (5)$$

The objective function can be iteratively reduced, or nonincreasing, via the following updating rules,

$$H = H .* (W^T X) ./ (W^T W H) \quad (6)$$

$$W = W .* (X H^T) ./ (W H H^T) \quad (7)$$

where $.*$ and $./$ denotes the element-wise multiplication and division between a pair of matrices, respectively.

The two matrices after factorization have the effect of indicating the cluster membership. The cluster membership c_i of the i -th interaction behavior is simply given by:

$$c_i = \underset{j}{\operatorname{argmax}} W_{ij} \quad (8)$$

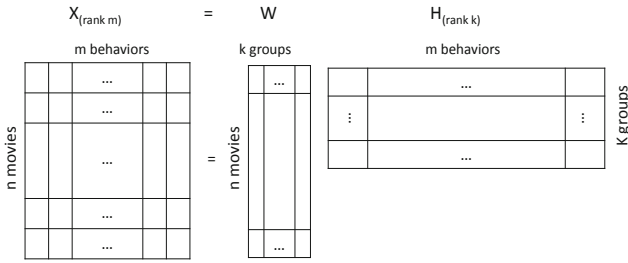


Fig. 5. A rank- k reduction of a behaviors of n movies and m behaviors is obtained by NMF , $X = WH$

where j is the label of the latent cluster of the interaction behaviors. Usually the number of latent clusters are a much smaller number than the total number of interaction behaviors.

In addition, the singular value decomposition (SVD) [11] was used we used to initialize the standard NMF algorithm. The initialization based on SVD enables NMF to perform better results compared to random initialization.

Model Selection. We computed NMF with different values of k (number of clusters). Then, for any rank k , NMF algorithm groups the samples into clusters. Therefore, we should determine the optimal k which decomposes the samples into ‘meaningful’ clusters. For this purpose we investigated ‘Homogeneity-Separation’ to optimize the number k of clusters, since that the standard definition of a good clustering is that of ‘homogeneity and separation’. Every element in a cluster must be highly similar (homogeneous) to the other elements in the same cluster and highly dissimilar (separation) to elements outside its own cluster. In addition, there are typically interactions between homogeneity and separation - usually, high homogeneity is linked with low separation, and vice versa.

5 Experimental Results

Table 5 shows the best solutions of behavior signals clustering for the ‘Homogeneity-Separation’ method. To further illustrate the advantage of the proposed method, we compare our method with the k-means clustering method. In order to understand the developmental similarity of the different groups of children, we compared both mental retardation infants and autistic infant with typical developmental infants.

Therefore, to calculate the disagreement between the 3 groups of children, we used the value of the Normalized Mutual Information (NMI) as proposed in [12]. The normalized mutual information of two different clusterings measures the agreement between the two clusterings \hat{y}^1 and \hat{y}^2 . Formally, the normalized mutual information of two clusterings \hat{y}^1 and \hat{y}^2 can be defined as:

Table 5. Number of obtained cluster by group of children

	Method	Typical	Autistic	Mental retardation
<i>Semester 1</i>	NMF	11	12	5
	K-means	12	8	7
<i>Semester 2</i>	NMF	14	8	11
	K-means	14	11	11
<i>Semester 3</i>	NMF	9	10	14
	K-means	12	12	10

$$NMI(\hat{y}^1, \hat{y}^2) = \frac{\sum_{i=1}^k \sum_{j=1}^k n_{i,j}^{1,2} \log \left(\frac{n \times n_{i,j}^{1,2}}{n_i^1 \times n_j^2} \right)}{\sqrt{\left(\sum_{i=1}^k n_i^1 \log \frac{n_i^1}{n} \right) \left(\sum_{j=1}^k n_j^2 \log \frac{n_j^2}{n} \right)}} \quad (9)$$

where n_i^1 is the number of behaviors assigned to the cluster label c_i in \hat{y}^1 , n_j^2 is the number of behaviors assigned to the cluster label c_j in \hat{y}^2 , and $n_{i,j}^{1,2}$ is the number of behaviors assigned to the cluster label c_i in \hat{y}^1 and c_j in \hat{y}^2 .

Table 6 shows the normalized mutual information values between the clustering result of typical infants/autistic infants, typical infants/mental retardation infants and autistic infants/mental retardation infants. The results show that the behaviors similarity between typical development children and autistic children is quite similar to the correlation between typical development children and mental retardation children during the first semester of life. However, *NMF* performs better than K-means, since that we obtained similarity between TD/AD equals to 0.482 during the first semester and 0.438 using *NMF* clustering method compared to 0.343 for semester 1 and 0.240 for semester 2 using k-means clustering method. From the clinical observation, *NMF* provides better results, since that signs of autism are not very obvious during the first year of life, the same things by comparing mental retardation and typical development children.

Using *NMF* clustering method, TD groups and MR groups are more correlated than TD groups and AD groups during the second and the third semester of age. On the other hand, the mutual information between TD second semester and MR third semester is about 0.522 using *NMF* method which explains the mental retardation of MR children compared to TD children. However, the behaviors similarity between typical development children and autistic children decreases during the time from the first semester to the third semester which is explained by the development deviance of autistic children. This is the quality that most differentiates the symptoms of autism from those of mental retardation [13].

Several studies on early home movies [14] have revealed that, during first year, mental retardation and autistic infants are characterized by a reduced response to the infant's own name, a reduced quality of affect and a tendency to look less to others. However, these pre-autistic signs become more obvious for autistic children. These characteristics are confirmed by the results obtained using *NMF* clustering method. Table 6 shows that, during the two first semesters, AD groups

Table 6. Normalized mutual information between TD/AD , TD/MR et AD/MR

	Mthode	TD/AD	TD/MR	AD/MR
$S1$	NMF	0.482	0.491	0.473
	$K\text{-moyennes}$	0.343	0.301	0.485
$S2$	NMF	0.438	0.520	0.506
	$K\text{-moyennes}$	0.240	0.262	0.399
$S3$	NMF	0.372	0.456	0.465
	$K\text{-moyennes}$	0.273	0.219	0.416

are quiet similar to MR groups (semester 1: 0.473; semester 2: 0.506). However, we obtained similarity equal to 0.399 using K-means clustering method.

In addition, preliminary results show that, using NMF clustering method, the behaviors dyads initiated by $G\text{-reu}$, regulation up, are always grouped together for autistic children group. This result is motivated by the interaction quality of the autistic children, then the parents act always to excite the child and to attract his attention.

6 Conclusions

In this paper, we propose a statistical data characterization based on $tf\text{-idf}$ representation which allows the automatic processing of parent-infant interaction behaviors. In addition, we propose the use of clustering method NMF to reduce the dimensionality of human (Parent/infant) behaviors in order to understand parent-infant interaction and to analyze the quality of parent-infant interactive behavior of mental retardation and autistic infant compared to typical developmental infant. Experimental results show that the developmental similarity of TD and AD groups decrease during the time. In addition we notice that the development of mental retardation infants is shifted compared to the normal development (TD).

Parent-infant interaction understanding is a multidisciplinary problem in which automatic analysis of interactions behaviors and the social psychology research are complementary [15]. Therefore, the next step of this work is to analyze with more details the clustering results (the behaviors contained by each cluster) and to investigate clinical interpretations with our psychologist partners to understand the interaction quality of each group of children.

Acknowledgments

Thanks to Filippo Muratori and Fabio Apicella from Scientific Institute Stella Maris of University of Pisa, Italy, who have provided data; family home movies. We would also like to extend our thanks to David Cohen and his staff, Raquel Sofia Cassel and catherine Saint-Georges, from the Department of Child and

Adolescent Psychiatry, AP-HP, Groupe Hospitalier Pitié-Salpêtrière, Université Pierre et Marie Curie, Paris France, for their collaboration and the manual database annotation and data analysis.

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