Tip-over stability-based path planning for a tracked mobile robot over rough terrains

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Most of the existing path planners for traversing over rough terrains use the single-valued probabilistic properties of the terrain with the extension of considering the robot's dimensions to build the cost function. The present work proposes a path planner for a tracked mobile robot to traverse over rough terrains using the robot's tip-over stability as its cost function. The contacts that the robot makes with the terrain determine the pose of the robot and in turn its tip-over stability. The estimation of the robot's pose is formulated as a linear complementary problem (LCP) and solved using the Lemke's method. We show some examples on searching paths that optimize for various cost functions over a randomly generated rough terrain. We also validate the performance of our pose estimator by comparing their results to those obtained from a dynamic simulator (MSC Adams).

Keywords: Tracked vehicle; mobile robot; path planning; rough terrain

1. Introduction

Tracked mobile robots are attractive for the terrestrial locomotion over rough terrains mainly for its large traction and stability due to the large contact area formed between the tracks and the terrain. Additionally, tracked mobile robots are mechanically robust due to their reduced number of degrees of freedom, and this aspect makes them be good candidates for traversing over rough terrains.

However, despite the traction, stability and robustness advantages that these robots boast, they also suffer from limitations. First, tracked mobile robots are unable to traverse over obstacles with height values superior to the radius of the tracks' sprocket[a]. In addition, these robots need to

[a]If flippers are added, its mobility is limited to the height that the flippers can reach.

laterally skid to generate rotational motions, and when they traverse over rough terrains, the effect of the sideslip might be magnified due to the irregularities of the terrain, potentially causing uncontrolled motions.

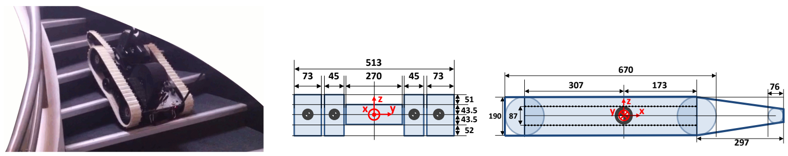
One way to overcome these limitations is to plan paths that avoid the aforementioned cases. The configuration space over which the paths are planned can not any longer have a binary representation (i.e., obstacle/free configuration spaces). In contrast, it needs to be represented using some continuous function that correlates with the traversability. The traversability may be defined as the product between the probability that the terrain slope is smaller than a chosen maximum permissible slope value and the probability that the roughness is smaller than a chosen maximum permissible roughness value.[1] It may also be defined as the sum of the roughness and the curvature (or slope) of a given grid cell known as *cell impedance*[2] or *traversability index*.[3] And, it may be defined as the probability that a robot pose satisfies the roll, pitch and height criteria for a wheeled microrover assuming that they have Gaussian distribution.[4] All these approaches use either single-valued probabilistic properties of the terrain and the robot's dimensions, or the interaction between the robot and the terrain but with fixed contact points (wheels).

In the present work, we estimate the traversability of the terrain for a tracked mobile robot as the tip-over stability computed by estimating its pose with no prior knowledge on how the robot will make contact with rough terrains. We believe that this approach gives a more realistic sense of traversability than the aforementioned probabilistic methods. Next, we search for paths that optimize this objective function using the A^* algorithm[5] to show the viability of our approach. Yet, the subject of the flipper motion planning will not be addressed in the present work.

The present work is organized as follows. We first describe our robot pose estimator over rough terrains in Section 2. Then, the definition of the tip-over stability used in the present work is given in Section 3. After, a description of the path planner is given in Section 4. Next, we present the simulation results and discuss about them in Section 5. Finally, in Section 6 we conclude by remarking the principal contributions of the present work and showing possible future work.

2. Estimating the pose of a tracked mobile robot over rough terrains

The tracked mobile robot that we modeled after is the *Cameleon EOD*[6] (Fig. 1(a)). It is 670 mm long, 513 mm wide and 190 mm high, and it



(a) *Cameleon EOD* (b) Front view (c) Side view

Fig. 1. A tracked mobile robot from *Eca Robotics*[6] and its model. All the values are in mm.

weighs about 27 kg. Its center of mass is 67 mm forward from its geometric center. The robot is modeled as a multi-rigid body where its main frame, the tracks and the flippers are modeled with their respective bounding boxes (Fig. 1(b) and Fig. 1(c)).

Estimating the pose of a tracked mobile robot over an rough terrain for a given triplet (x, y, γ) is a problem that consists of finding $\mathbf{q} = (z, \alpha, \beta)^T$, where $(x,\ y,\ z)$ are the Cartesian coordinates of the center of gravity of the robot and $(\alpha,\ \beta,\ \gamma)$ are the roll, pitch and yaw angles of the robot, respectively. (x, y, γ) will be given by the planner. The unknown vector $\mathbf{q}$ can be found by studying how the robot makes contact with the terrain. Therefore, the problem of estimating the robot's pose can be considered as a collision problem between the robot and the terrain. In the present work, we formulate this collision problem as a linear complementary problem (LCP) because of its fast convergence[7] and because this method seems to be appropriate for applications which involve sustained contacts.[8]

The equations of motion for estimating the robot's pose over an rough terrain can be formulated as follows

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{W_n}\mathbf{A}\mathbf{f} + \mathbf{Q_{co}} \tag{1}$$

$$\mathbf{M} = \mathrm{diag}(m, I_\alpha, I_\beta), \quad \mathbf{A} = \mathrm{diag}(a_1, a_2, \cdots, a_p), \quad \mathbf{f} = (f_1, f_2, \cdots, f_p)^T$$

where $\mathbf{M}$ is the 3×3 diagonal inertial matrix, (m, I_α, I_β) are the body mass, the moments of inertia about the roll axis and about the pitch axis, $\ddot{\mathbf{q}}$ is $(\ddot{z}, \ddot{\alpha}, \ddot{\beta})^T$, $\mathbf{W_n}$ is the $3 \times p$ Jacobian or Wrench matrix that maps normal contact forces to wrenches in the robot's body frame (where p is the total number of potential contact points), $\mathbf{f}$ is the $p \times 1$ vector of the contact forces along the normal direction, and $\mathbf{Q}_{co}$ is the 3×1 generalized conservative force (gravitational force). $\mathbf{A}$ is the $p \times p$ activation matrix that indicates whether each of the potential contact points are active (i.e., in contact with the terrain) with

$$a_i = \begin{cases} 1, & \text{if the } i\text{-th potential contact point collided} \\ 0, & \text{otherwise} \end{cases}$$

In (1) we only consider the non-penetration constraint since (x, y, γ) is fixed by the planner. Possible slippage due to the inclination of the terrain is considered in the path planning procedure as a friction violation constraint. In addition, we assume that the flippers are always parallel to the robot's main base, and the problem of flipper motion planning is left for a future work.

Two unknown vectors are present in (1): $\ddot{\mathbf{q}}$ and $\mathbf{f}$. However, $\mathbf{f}$ can be solved by noticing the complementary relationship for each potential contact point between its contact force and its distance from the terrain. This complementary problem is linear and can be formulated as

$$\mathbf{0} \leq \mathbf{d} \perp \mathbf{f} \geq \mathbf{0} \tag{2}$$

where $\mathbf{d} = (d_1, \cdots, d_p)^T$ are the distance between the potential contact points on the robot's body and the projection of these points on the terrain along the vertical direction, $\mathbf{f}$ is defined in (1), and $\perp$ is the complementarity operator. (2) says that for any i, if d_i is zero then f_i must be positive, and if d_i is positive then f_i must be zero.

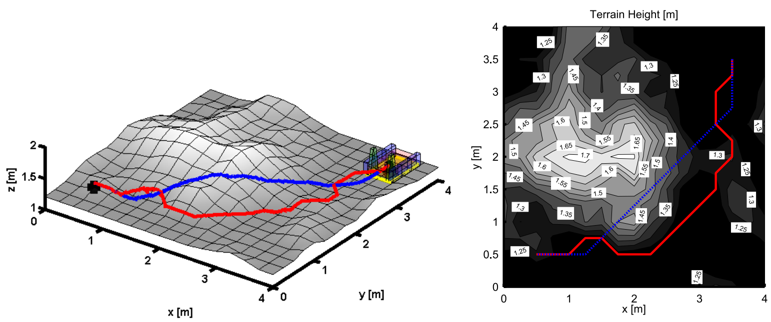
The linear complementary problem (2) is solved for the contact force vector $\mathbf{f}$ using the Lemke's method.[9] Therefore, $\mathbf{f}$ becomes known by solving (2), and, finally, $\ddot{\mathbf{q}}$ can be found by solving (1).

3. Tip-over stability

In the present work, we use the tip-over stability of the robot as part of the cost function for planning paths over rough terrains. The force-angle algorithm[10] is used to estimate the robot's tip-over stability because of its proved good performance.[11]

First, the net force acting on the robot's center of gravity ($\mathbf{f_r}$) is computed. Assuming that the speed of the tracked mobile robot is small as it moves over rough terrains, the quasi-static stability is considered. Therefore, the only force acting on the robot's center of gravity is the gravitational force. Next, for each edge that form the convex hull, its normal vector that intersects with the robot's center of gravity is searched. Then, the angle formed between each of these normal vectors and the net force vector is computed (θ_i). Finally, the force-angle (α) is computed as the product between $\min\limits_i (\theta_i)$ and the magnitude of the net force ($\mathbf{f_r}$). Then, this measure is normalized by its maximum value as follows

$$\hat{\alpha} = \frac{\min\limits_i (\theta_i) \|\mathbf{f}_r\|}{\alpha_{\max}}, \quad i = \{1, \cdots, n\}. \tag{3}$$



(a) (b)

Fig. 2. Path planning for traversing over a rough terrain. (a) Elevation map of a rough terrain with the *path* ω_1 (blue dotted curve) and the *path* ω_2 (red curve) (b) Contour of the elevation map with the *path* ω_1 (blue dotted curve) and the *path* ω_2 (red curve).

where n is the number of the edges that form the convex hull. The positive and negative values of $\hat{\alpha}$ indicate stable and unstable configurations in the tip-over stability sense, respectively. The robot is critically stable when $\hat{\alpha} = 0$.

4. Path planning algorithm

The A* algorithm[5] is used to show the viability of our approach to plan path from the estimate of the robot's tip-over stability over a randomly generated rough terrain. The configuration space consists of (x, y, γ), where (x, y) are the horizontal coordinates of the robot in the inertial frame, and γ is the yaw angle of the robot, which can only have one of the following eight values: $\gamma = i \cdot \frac{\pi}{4},\ i = \{0, 1, \cdots, 7\}$.

The A* is a graph-based algorithm that searches a minimum-cost path from a given start node to a given goal node. The cost of a new node has the following expression

$$Cost = \omega_g \cdot g + \omega_h \cdot h \tag{4}$$

where g is the cost from the start node to the new node, h is a heuristic function that estimates the cost from the new node to the goal node, and ω_g and ω_h are respective weights. In turn, g is defined as

$$g = g_{\text{prev}} + \omega_\alpha \cdot (1 - \hat{\alpha}) + \omega_\gamma \cdot \frac{\Delta\gamma}{\Delta\gamma_{\max}} + \omega_\ell \cdot \frac{\ell}{\ell_{\max}} \tag{5}$$

where $(\hat{\alpha}, \Delta\gamma, \Delta\gamma_{\max}, \ell, \ell_{\max})$ are the normalized tip-over stability measure, the change in yaw angle, the maximum change in yaw angle, the distance between the two configurations, and the maximum distance between any two

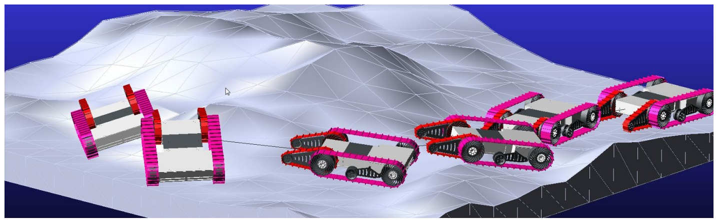


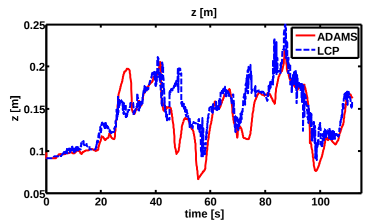
Fig. 3. Snapshots of tracking the path ω_2 using MSC Adams.

neighboring configurations, respectively. On the other hand, $(\omega_\alpha, \omega_\gamma, \omega_\ell)$ are the weights corresponding to the tip-over stability measure, the change in yaw angle and the distance between the two configurations, respectively. h consists of the Euclidean distance between the new node and the goal node.

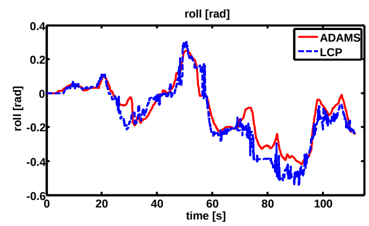
5. Results and discussion

We first show the performance of our tip-over stability-based path planner, which is implemented using Matlab. As shown in Fig. 2, the robot is asked to traverse over a rough terrain. We considered two sets of cost weights obtaining two different paths: $\boldsymbol{\omega_1} = (\omega_g, \omega_h, \omega_\alpha, \omega_\gamma, \omega_\ell) = (0.999, 0.001, 1/3, 1/3, 1/3)$ and $\boldsymbol{\omega_2} = (\omega_g, \omega_h, \omega_\alpha, \omega_\gamma, \omega_\ell) = (0.999, 0.001, 1, 0, 0)$. $\boldsymbol{\omega_1}$ equally considers the tip-over stability measure, the cost for yaw angle changes and the Euclidean distance between two consecutive configurations, while $\boldsymbol{\omega_2}$ considers only the tip-over stability measure. For both cases the weight of the heuristic function is low. Let us call the paths generated with $\boldsymbol{\omega_1}$ and $\boldsymbol{\omega_2}$ as the *path* ω_1 and *path* ω_2, respectively. Fig. 2(b) shows that the *path* ω_1 corresponds to the blue dotted curve indicating the location of the robot's center of gravity, and the *path* ω_2, the red-colored curve. We see that the *path* ω_2 reaches the goal circumvallating hazardous regions by choosing configurations with high tip-over stability measure, while the *path* ω_1 could not avoid dangerous regions due to additional considerations of the costs associated with yaw angle changes and the Euclidean distance between configurations.

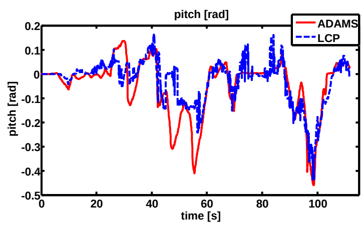
Further, we generate a trajectory from the path that optimizes (4) and track the resulting trajectory using a dynamic simulator (MSC Adams) (Fig. 3). In the simulator, the flexible toothed belts on the tracks are modeled as a series of wheels rotating at the same velocity as the sprocket, which allows the simulator to efficiently integrate the robot motion. In addition, the wheel-ground contact stiffness values along the robot are tuned



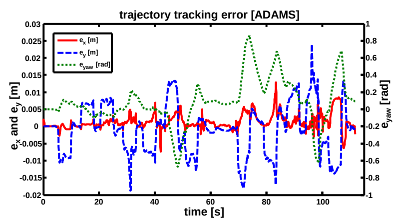
(a) height



(b) roll



(c) pitch



(d) tracking error

Fig. 4. (a) - (c) Comparison results between our pose estimator and MSC Adams while the robot is asked to track the path ω_2 for traversing over the rough terrain shown in Fig. 2. (d) forward, lateral and yaw errors obtained from MSC Adams.

such that the loads are mainly concentrated underneath the active and passive sprockets and rollers. The trajectory tracking controller used in the simulator is a kinematic controller based on the unicycle model. The controller tracks the reference trajectory by controlling an unconstrained point located on the robot, which is shifted forward from the robot's center of gravity to track the trajectory. The main drawback of this controller is its inability to control the robot's orientation accurately. In the future, a more robust controller will be developed.

In Fig. 4, we compare the height, roll and pitch values obtained from our pose estimator to those obtained from MSC Adams while the robot is asked to track the path ω_2 for traversing over the rough terrain shown in Fig. 2. In general, the height-roll-pitch results obtained from both the pose estimator and MSC Adams are close to each other except for the cases when the tracking error becomes significant in yaw, mainly due to the limitation of our controller. The presented results suggest that when the robot tracks the desired path well, then the proposed pose estimator gives reliable results.

6. Conclusion and future work

A path planner using the robot's tip-over stability as the cost function is proposed for a tracked mobile robot to traverse over rough terrains using the A* algorithm. We present results on the performance of both the path planner and the pose estimator over a randomly generated rough terrain. The path planner is able to avoid dangerous regions and choose stable configurations to reach the goal. The results obtained from our pose estimator are in general close to those obtained from a dynamic simulator, except when significant trajectory tracking error is present. In the near future, a better tracking controller will be studied. In addition, a randomized planner will be used for a higher planning efficiency. Finally, the presented work will be implemented on an actual tracked mobile robot (Fig. 1(a)).

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