Dynamic Coupling Map: Trajectory Analysis Technique for Dynamic Motions of Underactuated Systems

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Humans and animals are capable of overcoming complex terrain challenges with graceful and agile movements. One of the key ingredients for such complex behaviors is motion coordination to exploit their natural dynamics. Sports performers coordinate their action in many different ways to achieve their goals. Coordination is a key feature from the graceful, precise action of an ice dancer to the explosive, physical power of a triple jumper. Lizard coordinate its tail swing to stabilize its dynamic motion over rough terrain [4]. Cheetah can rapidly accelerate and maneuver during the pursuit of its prey by the coordinating of the motion of its tail [7]. Understanding and emulating these motions is the one of the long-standing grand challenges in robotics and biomechanics,with possible applications in rehabilitation, sport, search-and-rescue, environmental monitoring and security.

Synthesizing motion behavior for such underactuated systems is quite challenging. Underactuation impose constraints on their dynamics that restrict the family of trajectories their configurations and accelerations can follow. These constraints are second-order nonholonomic constraints [12][5]. Moreover, cannot be fully feedback linearized [9]. Underacuated system can only be controlled indirectly either through contact forces with the environment or through inertial forces which rises from the nonlinear inertial coupling of the articulated system [10], in the presented work we focus on the latter.

Swing-up and throwing tasks for underactuated manipulators are examples of dynamic motions that exhibit highly nonlinear coupling dynamics. During the last two decades, many researchers have investigated the approach of passivity or energy-based control of underactuated mechanical systems.[3] showed that the coupling becomes more complex when the number of links increases, and its control problem becomes more challenging. [13] noted that energy based techniques are difficult to apply for higher dimension robots. In this work, we employ nonlinear trajectory optimization for the swing-up motion generation and control. Trajectory optimization approach is becoming increasingly attractive with the advent of computational power and the recent advances in nonlinear optimization[8][2][6]. Given an initial trajectory that may be non-optimal or even non-feasible, trajectory optimization methods can often quickly converge to a high-quality, locally-optimal solution. Furthermore, there exists strong evidence that humans solve task-level and motor-level challenges though optimization processes [11].

Despite the existence of powerful tools such as nonlinear trajectory optimization, they are usually treated as blackboxes that provide local optimal trajectories. We introduce the "*Dynamic Coupling Map (DCM)*", a novel graphical technique, to help gain insight into the output trajectory of the optimization and analyze the capability of underactuated robots. The DCM analysis is demonstrated on the swing up motion of a simplified model of a gymnast on high bar. The DCM shows in a graphical and intuitive way the pivotal role of exploiting the nonlinear inertial forces to reach the unstable equilibrium configuration while taking into account the torque bounds constraints. In this work, we present the DCM as a posteriori analysis of a local optimal trajectory, found by employing the direct collocation trajectory optimization framework.

For a test case scenario, we adopted the swing-up motion of "Gymnast robot" as shown in figure (1). The local optimal trajectory found using SNOPT [1] is shown in figure (2). Finally, the DCM shown in figure (3) highlights the importance of exploiting the non-linear inertial coupling to achieve such dynamic motion.

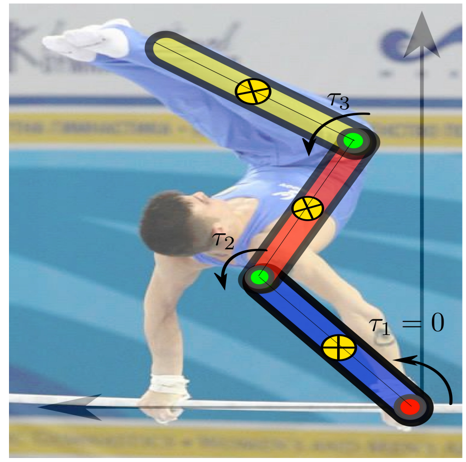

Figure 1: Simplified model of a gymnast on a high bar

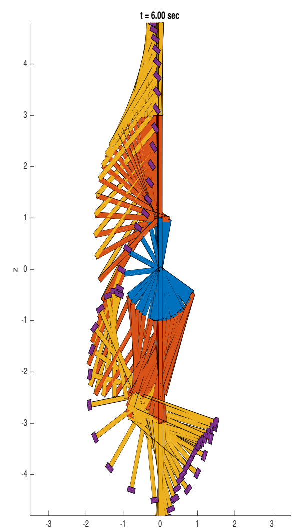

Figure 2: Local-optimal swing-up motion

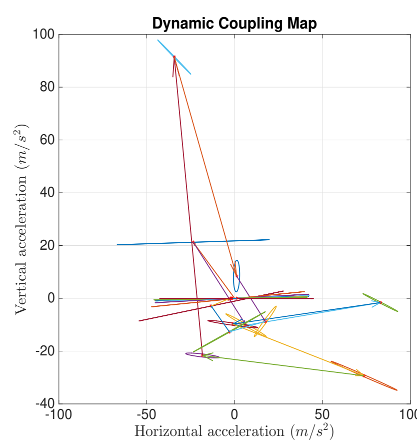

Figure 3: Dynamic Coupling Map of the swing-up motion for the gymnast robot

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