# A new method for determining the location of the instantaneous axis of rotation during human movements 

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## 1 Introduction

An accurate location of the instantaneous axis of rotation (IAR) between body segments is of primary importance for biomechanical applications, in particular for accurate estimation of muscle forces/torques. Among the numerous studies that addressed this issue, the International Society of Biomechanics recommended the instantaneous helical axis (IHA) method, which considers that the transition from one position to another for the same solid can be obtained by a rotation around an axis followed by a translation along this axis. Nevertheless, this has the serious drawback of being very sensitive to low angular velocity [1].
The present study proposes a new method for the determination of the IAR during human movements. Our specific aim is to provide an easy-to-implement method, suitable for all types of movement and whatever the angular velocity of motion.

## 2 Methods

The mathematical formulation of the method will be tested firstly by way of simulations then within the context of human movements.

### 2.1 Mathematical formulation of the method

Suppose an absolutely rigid body $S$. There is an axis $\Delta$ such as $\Delta=(P, \vec{\Omega})$ where the rigid body $S$ turns and translates relatively to a fixed frame $\mathrm{R}_{0}$ among this axis as illustrated figure 1.
Let be P , a point fixed on $\Delta$, then:

$$
\begin{equation*}
\vec{V}_{\left(P \in S / R_{0}\right)}=\lambda \cdot \vec{\Omega}_{S / R_{0}} \tag{1}
\end{equation*}
$$

With:

$$
\begin{equation*}
\vec{V}_{\left(A \in S / R_{0}\right)}=\vec{V}_{\left(P \in S / R_{0}\right)}+\vec{\Omega}_{S / R_{0}} \wedge \overrightarrow{P A} \tag{2}
\end{equation*}
$$



Figure 1: Relative movement between a solid $S$ and a fixed frame $R_{0}$ around an axis.
Our method consists of the determination of the location of $\Delta$ which represent the IAR.
Let multiply (1) by $\vec{\Omega}_{S / R_{0}}$, then (2) becomes:

$$
\begin{equation*}
\vec{\Omega}_{S / R_{0}} \wedge \vec{V}_{\left(A \in S / R_{0}\right)}=\vec{\Omega}_{S / R_{0}} \wedge\left(\vec{V}_{\left(P \in S / R_{0}\right)} \wedge \overrightarrow{P A}\right) \tag{3}
\end{equation*}
$$

with: $\vec{\Omega}_{S / R_{0}} \wedge \vec{V}_{\left(P \in S / R_{0}\right)}=\overrightarrow{0}$ because $\vec{\Omega}_{S / R_{0}}$ and $\vec{V}_{P \in S / R_{0}}$ are collinear, as it is write in (1).
Let be $\overrightarrow{P A}=\overrightarrow{P A}+\overrightarrow{A^{\prime} A}$ in (3) where $A^{\prime}$ is the orthogonal projection of $A$ on, we obtain:

$$
\begin{equation*}
\overrightarrow{A A}^{\prime}=\frac{\vec{V}_{\left(A \in S / R_{0}\right)} \wedge \vec{\Omega}_{S / R_{0}}}{\left(\vec{\Omega}_{S / R_{0}}\right)^{2}} \tag{4}
\end{equation*}
$$



Figure 2: Relative movement between two rigid bodies S1 and S2.

Let apply this to the relative movement between two rigid bodies $S 1$ and $S 2 . A_{i} \in S 1$ and $B_{j} \in S 2$
are certain fixed points on the system and $\mathrm{A}_{\mathrm{i}}^{*}$ and $B_{j}^{*}$ are respectively the orthogonal projections of $A_{i}$ and $B_{j}$ points on the IAR between the two bodies as illustrated figure 2 . For this example, the equation (3) becomes:

$$
\left\{\begin{array}{l}
\overrightarrow{\mathrm{A}_{\mathrm{i}} \mathrm{~A}_{\mathrm{i}}^{*}}=\frac{\overrightarrow{\mathrm{V}}_{\left(\mathrm{A}_{\mathrm{i}} \in \mathrm{~S} 1 / \mathrm{S} 2\right)^{\wedge} \wedge} \vec{\Omega}_{\mathrm{S} 1 / \mathrm{S} 2}}{\left(\vec{\Omega}_{\mathrm{S} 1 / \mathrm{s} 2}\right)^{2}}  \tag{5}\\
\overrightarrow{\mathrm{~B}_{\mathrm{j}} \mathrm{~B}_{\mathrm{j}}^{*}}=\frac{\overrightarrow{\mathrm{V}}_{\left(\mathrm{B}_{\mathrm{j}} \in \mathrm{~S} 2 \mathrm{~S} 1\right) \wedge} \vec{\Omega}_{\mathrm{S} 2 / \mathrm{s} 1}}{\left(\vec{\Omega}_{\mathrm{S} 2 \mathrm{~S} 1}\right)^{2}}
\end{array}\right.
$$

$\vec{V}_{\left(\mathrm{A}_{\mathrm{i}} \in \mathrm{S} 1 / \mathrm{S} 2\right)}$ could be known using composition of movement:
$\vec{V}_{\left(\mathrm{A}_{\mathrm{i}} \in \mathrm{S} 1 / \mathrm{S} 2\right)}=\vec{V}_{\left(\mathrm{A}_{\mathrm{i}} \in \mathrm{S} 1 / \mathrm{R}_{0}\right)}-\vec{V}_{\left(\mathrm{A}_{\mathrm{i}} \in \mathrm{S} 2 / \mathrm{R}_{0}\right)}$
And property of rigid body:
$\vec{V}_{\left(A_{i} \in S 2 / R_{0}\right)}=\vec{V}_{\left(B_{j} \in S 2 / R_{0}\right)}+\vec{\Omega}_{S 2 / R_{0}} \wedge{\vec{B} \vec{A}_{i}}$
Applying the same computation for $\vec{V}_{\left(B_{j} \in \mathrm{~S} 2 / \mathrm{S} 1\right)}$, (5) becomes:
$\left\{\begin{array}{l}\overrightarrow{A_{i} A_{i}^{*}}=\frac{\left(\vec{V}_{\left(A_{i} \in S 1 / R_{0}\right)}-\vec{V}_{\left(B_{j} \in S 1 / R_{0}\right)}-\vec{\Omega}_{S 2 / R_{0}} \wedge{\left.\overrightarrow{B_{j}} \vec{A}_{\mathrm{i}}\right)}\right) \vec{\Omega}_{\mathrm{S} 1 / \mathrm{S} 2}}{\left(\vec{\Omega}_{\mathrm{S} 1 / \mathrm{S} 2}\right)^{2}} \\ \overrightarrow{\mathrm{~B}_{\mathrm{j}} \mathrm{B}_{\mathrm{j}}^{*}}=\frac{\left(\vec{V}_{\left(\mathrm{B}_{\mathrm{j}} \in \mathrm{S} 2 / R_{0}\right)}-\vec{V}_{\left(\mathrm{A}_{\mathrm{i}} \in \mathrm{S} 2 R_{0}\right)}-\vec{\Omega}_{\mathrm{S} 1 / R_{0}} \wedge{\left.\overrightarrow{A_{i}} \overrightarrow{\mathrm{~B}}_{\mathrm{j}}\right) \vec{\wedge} \vec{\Omega}_{\mathrm{S} 2 / \mathrm{S} 1}}_{\left(\vec{\Omega}_{\mathrm{S} 2 \mathrm{~S} 1}\right)^{2}}\right.}{}\end{array}\right.$
We easily obtain the coordinates of each $A_{i}^{*}$ and $B_{j}^{*}$ points. All $A_{i}^{*}$ and $B_{j}^{*}$ points should be aligned between each other and allow the location of the IAR. In order to check the alignment of this points, the coordinates of the $A_{i}^{*}$ and $B_{j}^{*}$ points were fitted by linear regression.

### 2.2 Simulations

The ability of the proposed method to provide accurate location of the IAR was first evaluated through simulations by considering the case of the motion of two homogeneous rigid bodies linked by frictionless hinges. The relative motion of these two bodies with a random trajectory and at different angular velocity were simulated on Matlab.

### 2.3 Experimental analysis

A 3 cx1 units Codamotion system (Charnwood Dynamics Ltd., Rothley, United Kingdom) was used to collect kinematic data at 400 Hz during wrist flexion-extension movements performed at different speeds (slow, normal fast). Eight active markers ( 4 on the hand and 4 on the forearm) were placed on the body.

## 3 Results and Discussion

### 3.1 Simulation results:

Figure 3 represents the location of the IAR fitted by linear regression at one instant of the simulation. The mean determination coefficients
(R2) obtained between the coordinates $x$ and $y$ then $x$ and $z$ then $y$ and $z$ of the $A_{i}^{*}$ and $B_{j}^{*}$ points (Table 1) indicated very high accuracy of the proposed method for the computation of the location of the IAR. As expected from the new formulation of our method, results showed that this method is not sensitive to low angular velocity.


Figure 3: Determination of the IAR.
Table1. Determination coefficient

|  | $(\mathrm{x}, \mathrm{y})$ | $(\mathrm{x}, \mathrm{z})$ | $(\mathrm{y}, \mathrm{z})$ |
| :---: | :---: | :---: | :---: |
| R2 simulation | $1 \pm 0$ | $1 \pm 0$ | $1 \pm 0$ |
| R2 experimental | $0.89 \pm 0.4$ | $0.88 \pm 0.3$ | $0.91 \pm 0.6$ |

### 3.2 Experimental results:

The mean determination coefficients (R2) obtained between the coordinates x and y then x and $z$ then $y$ and $z$ of the $A_{i}^{*}$ and $B_{j}^{*}$ points (Table 1) show that each $A_{i}^{*}$ and $B_{j}^{*}$ points are not totally aligned between them. Even if the alignment of $A_{i}^{*}$ and $B_{j}^{*}$ point is not perfect, the method provides a good estimation of the location of the IAR without sensitivity to low angular velocity. Current work will lead to an improvement of the experimental results.

## 4 Conclusions

The present study proposes a new method to determine the location of the IAR during human movements. Simulations results demonstrated its easiness to implement with low computational requirements and its high accuracy with low sensitivity to angular velocity. Despite limitations which are presently addressed, the experimental results are encouraging and will find direct applications for biomechanical modelling and for identification of inertia parameters.

## References

[1] Ehrig RM, Taylor WR, Duda GN, Heller MO. 2006. A survey of formal methods for determining the centre of rotation of ball joints. J Biomech. 39: 2798-2809.

